STUDY OF $\pi N \to \pi\pi N$ PROCESSES ON POLARIZED TARGETS II: THE PREDICTION OF $\rho^0(770) - f_0(980)$ SPIN MIXING

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Abstract

In Part I. of this work we have presented evidence that the measured relative phases of transversity amplitudes in $\pi N \to \pi\pi N$ processes differ from those predicted by the unitary evolution law. We ascribed this difference to a non-unitary interaction of the produced final state $\rho_f(S)$ with a universal quantum environment in the physical Universe. This new kind of interaction must be a pure dephasing interaction in order to render the S-matrix dynamics of particle scattering processes accessible to experimental observation. If the quantum environment is to be an integral part of the Nature then its dephasing interactions must be fully consistent with the Standard Model.

In this work we impose on the dephasing interaction the requirements of the conservation of the identities of the final state particles including their four-momenta, Lorentz symmetry, P-parity and the conservation of total angular momentum and isospin. From this consistency alone we find that the dephasing interaction must be a dipion spin mixing interaction. The observed amplitudes are a unitary transform of the corresponding S-matrix amplitudes. The elements of the spin mixing matrix are forward scattering amplitudes of dipion spin states with recoil nucleon spin states with initial and final dipion spins J and $K = J-1, J, J+1$, respectively. Helicities are conserved in this scattering. These amplitudes are matrix elements of four Kraus operators describing the non-unitary interaction with the environment. The theory predicts $\rho^0(770) - f_0(980)$ mixing in the S-and P-wave amplitudes in $\pi^- p \to \pi^-\pi^+ n$. The predicted moduli and relative phases of the mixed amplitudes are in an excellent qualitative agreement with the experimental results. The spin mixing of S-matrix partial wave amplitudes to form new observable partial wave amplitudes is a new phenomenon beyond the Standard Model. It is our conjecture that the non-unitary pure dephasing interaction describes the non-standard interaction of the baryonic matter with dark matter and dark energy which we identify with the quantum environment.

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Contents

I. INTRODUCTION.

Following the discovery in 1961 of ρ meson in $\pi N \to \pi\pi N$ reactions, the measurements of forward-backward asymmetry in $\pi^- p \to \pi^-\pi^+ n$ suggested the existence of a rho-like resonance in the S-wave amplitudes, later referred to as $\sigma(750)$ scalar meson [1–5]. The CERN measurements of $\pi^- p \to \pi^-\pi^+ n$ and $\pi^+ n \to \pi^+\pi^- p$ on polarized targets in 1970's cofirmed the existence of $\sigma(750)$ scalar meson. Evidence for a narrow $\sigma(750)$ was found in amplitude analyses of $\pi^- p \to \pi^-\pi^+ n$ at 17.2 GeV/c [6–13] and in $\pi^+ n \to \pi^+\pi^- p$ at 5.98 and 11.85 [11–13]. Additional evidence for $\sigma(750)$ came from the amplitude analysis of the ITEP data on $\pi^- p \to \pi^-\pi^+ n$ on polarized target at 1.78 GeV/c [14]. These findings were controversial because the measurements of $\pi^- p \to \pi^0\pi^0 n$ at CERN in 1972 found no evidence for a rho-like meson in the S-wave amplitudes [15]. In 2001, E852 Collaboration at BNL reported high statistics measurements $\pi^- p \to \pi^0\pi^0 n$ at 18.3 GeV/c [16]. Once again, there was no evidence for a rho-like resonance in the $\pi^0\pi^0$ S-wave amplitudes.

The resolution of the puzzle of $\sigma(750)$ resonance came from our recent high resolution amplitude analysis of the CERN data on $\pi^- p \to \pi^-\pi^+ n$ for dipion masses 580-1080 MeV at low momentum transfers [17, 19] where S- and P-wave amplitudes dominate. The analysis shows that the rho-like resonance in the S-wave transversity amplitudes arises entirely from the contribution of the $\rho^0(770)$ resonance to the data component in the expression for the moduli of the S-wave amplitudes. The S-wave transversity amplitudes are nearly either in phase or 180° out of phase with the resonating P-wave transversity amplitudes. These facts allow us to identify $\sigma(750)$ with $\rho^0(770)$. There is a pronounced dip at 980 MeV in the moduli of P-wave amplitudes $|L_d|^2$ that is identified with $f_0(980)$ resonance leading to $\rho^0(770) - f_0(980)$ mixing in both S- and P-wave amplitudes. A model independent determination of helicity amplitudes from the transversity amplitudes shows a $\rho^0(770)$ peak in the S-wave single flip helicity amplitude $|S_1|^2$ [19]. The phase of the amplitude S_1 is close to the phase of $\rho^0(770)$ Breit-Wigner amplitude [18, 19].

Identifying $\sigma(750)$ with $\rho^0(770)$ explains why there is no rho-like resonance observed in $\pi^- p \to \pi^0\pi^0 n$ since there is no P-wave amplitude in this process. However, the $\rho^0(770) - f_0(980)$ mixing appears to violate rotational and Lorentz symmetry. If this symmetry were violated in strong interactions we should observe a dependence of $\rho^0(770)$ width Γ_ρ on its helicity λ. In our analysis [17, 19] we demonstrate the independence of Γ_ρ on the helicity confirming the rotational and Lorentz symmetry of strong interactions. This suggests that the observed $\rho^0(770) - f_0(980)$ mixing arises from a new kind of interaction independently involved in the pion creation process.

The observed $\rho^0(770) - f_0(980)$ mixing can be naturally explained when we assume that the produced partial waves are not isolated spin states but interact with a common environment. We can see how this might work using the following mechanical analogy. Consider a pendulum of mass m and length L oscillating with a natural frequency ω, and an ensemble of several other penduli with various masses m_i and lengths L_i. When the other penduli are isolated from the oscillating pendulum they do not oscillate. However, when all the penduli are attached to a common rod all penduli begin to oscillate with the same resonant frequency although with different amplitudes. The interaction of the penduli with a common environment - the rod - allows a resonance from one pendulum to "leak" into the other penduli which have different natural frequencies. Similarly we can imagine that the partial waves produced in $\pi^- p \to \pi^-\pi^+ n$ process interact with a quantum environment which allows resonances from one partial wave amplitude to "leak" into other partial wave amplitudes. Then the pion creation process is no longer an isolated system but behaves as an open quantum system described by a non-unitary dynamics. A non-unitary evolution law must evolve the produced final state $\rho_f(S)$ into an observed final state $\rho_f(O)$ with new spin mixing partial wave amplitudes.

But how do we know that such quantum environment actually exists? And why its interaction with the $\pi^- p \to \pi^-\pi^+ n$ process should lead to the $\rho^0(770) - f_0(980)$ mixing?

The basic assumption of the S-matrix theory is that the Minkowski spacetime is empty. It contains no environment with which particle scattering processes could interact. In Part I. of this work [20] we put forward a hypothesis that the physical space is not empty but contains a universal quantum environment that can interact with some particle scattering processes. The new interaction cannot change the identity and the four-momenta of the final state particles produced by the S-matrix dynamics in order to leave the particle scattering dynamics open to the experimental observation. This means the new interaction must be a pure dephasing interaction that modifies phases of the amplitudes and nothing else. This is a new kind of interaction outside of the Standard Model.

To test our hypothesis we used unitarity of the S-matrix to obtain information on the relative phases of the measured partial wave transversity amplitudes $U^J_{\lambda\tau}$ and $N^J_{\lambda\tau}$. Unitary evolution law evolves pure initial states into pure final states. This fact implies that the relative phases are constrained to $0, \pm\pi, \pm 2\pi, ...$ in a disagreement with the measured phases in all analyses of $\pi N \to \pi\pi N$ processes. An amplitude analysis of the S- and P-wave subsystem in $\pi^- p \to \pi^-\pi^+ n$ assuming these unitary phases is in disagreement with the CERN data on polarized target at least at 5σ level. We conclude that the CERN measurements of $\pi^- p \to \pi^-\pi^+ n$ and $\pi^+ n \to \pi^+\pi^- p$ on polarized targets support the hypothesis of the existence of the quantum environment and its dephasing interaction with some particle scattering processes.

If the quantum environment is to be an integral part of the Nature then its dephasing interactions with particle scattering processes cannot contradict Standard Model. The two aspects of the Nature must be mutually consistent. What are the cosequences of this consistency?

The dephasing interaction evolves the produced S-matrix final state $\rho_f(S)$ into an observed final state $\rho_f(O)$. The non-unitary evolution law is described by Kraus representation. In this paper we show that the consistency of the pure dephasing interaction with the Standard Model in $\pi N \to \pi\pi N$ processes requires that it be a dipion spin mixing interaction. In what we call spin mixing mechanism the observed partial wave amplitudes are a unitary transform of the corresponding S-matrix partial wave amplitudes. The elements of the spin mixing matrix are forward scattering amplitudes of the partial waves with initial and final dipion spins J and $K = J-1, J, J+1$, respectively. Helicities are conserved in this scattering. These amplitudes are matrix elements of Kraus operators describing the non-unitary interaction with the environment. The theory predicts $\rho^0(770) - f_0(980)$ mixing in the S-and P-wave amplitudes in $\pi^- p \to \pi^-\pi^+ n$. The predicted moduli and relative phases of the mixed amplitudes are in an excellent qualitative agreement with the experimental results [17, 19]. The theory correctly predicts the absence of $f_2(1270) - f_0(1370)$ mixing.

The spin mixing of S-matrix partial wave amplitudes to form new observable partial wave amplitudes is a new phenomenon outside of Standard Model and represents a genuine new physics. It seems natural to connect this new physics with the physics of dark matter and dark energy. It is our conjecture that the non-unitary pure dephasing interaction describes the non-standard interaction of the baryonic matter with dark matter and dark energy which we identify with the quantum environment.

We discuss Kraus representation and Kraus operators in Section II. The non-unitary evolution of the produced final state $\rho_f(S)$ into an observed final state $\rho_f(O)$ is introduced in Section III. In Section IV. we use the purity of the dephasing interaction to derive the form of spin mixing mechanism and define Kraus partial wave helicity amplitudes which are mixtures of the S-matrix partial wave amplitudes. In Section V. the requirements of Lorentz symmetry, conservation of total angular momentum, P-parity and total isospin impose consistency constraints on the spin mixing mechanism and Kraus helicity amplitudes. In Section VI. self-consistency requirements constrain the spin mixing mechanism to a simple unitary transform of the S-matrix amplitudes. In Section VII. we relate the observed effective amplitudes to the Kraus amplitudes and demonstrate the

conservation of probability by the dephasing intaraction. In Section VIII. we discuss resonance mixing in $\pi^- p \to \pi^- \pi^+ n$ below 1400 MeV and calculate predictions for the moduli and phases of the mixed S- and P-wave amplitudes from the $U(2)$ transform below 1080 MeV describing the $\rho^0(770) - f_0(980)$ mixing. We conclude in Section IX. with a brief summary and a discussion of the physical nature of the quantum environment.

II. NON-UNITARY EVOLUTION OF OPEN QUANTUM SYSTEMS.

The interaction of an open quantum system S_i with a quantum environment E is assumed to be described by a unitary operator U. The initial state $\rho_i(S_i, E)$ of the combined system is prepared in a separable state $\rho_i(S_i, E) = \rho_i(S_i) \otimes \rho_i(E)$. The interaction evolves $\rho_i(S_i, E)$ into a final state [22–25]

$$\rho_f(S_f, E) = U \rho_i(S_i, E) U^+ \tag{2.1}$$

Unitary evolutions are entangling quantum operations and the joint state $\rho_f(S_f, E)$ is not a separable but an entangled state of the final system S_f and the environment E. The observer can perform measurements on the systems S_i and S_f but cannot perform direct measurements on the environment E. After the transformation U, the system S_f no longer interacts with the environment E. The quantum state of system S_f is then fully described by reduced density matrix

$$\rho_f(S_f) = Tr_E(\rho_f(S_f, E)) \tag{2.2}$$

in a sense that we can calculate average values $< \hat{O} >= Tr(\hat{O} \rho_f(S_f))$ of any observable $\hat{O}$. The trace in (2.2) is over the interacting degrees of freedom of the environment $|e_\ell >, \ell = 1, M$. In general, the reduced state $\rho_f(S_f)$ is no longer related to the initial state $\rho_i(S_i)$ by a unitary transformation $\rho_f = S \rho_i S^+$. Instead it is given by Kraus representation [22–25]

$$\rho_f(S_f) = \sum_{\ell=1}^{M} A_\ell \rho_i(S_i) A_\ell^+ \tag{2.3}$$

where we assumed that $\rho_i(E) = |e_0 >< e_0|$ is a pure state. The evolution operators $A_\ell =< e_\ell | U | e_0 >$ are called Kraus operators. They are acting on the Hilbert spaces $H(S_i)$ and $H(S_f)$ of the systems S_i and S_f, respectively, and can be unitary or non-unitary. For trace preserving maps the Kraus operators must satisfy completness relation

$$\sum_{\ell=1}^{M} A_\ell^+ A_\ell = I \tag{2.4}$$

To be physically meaningful, a linear map $\rho(S_i) \to \rho(S_f) = \$\rho(S_i)$ must be completely positive in order to preserve the positivity of all probabilities. A linear map $\$$ is completely positive if and only if it has the form of the Kraus representation [22–25]. The operator $\$$ is also called superscattering operator.

The quantum states $|e_\ell >$ of the environment form an orthonormal basis in Hilbert space $H(E)$ with a finite dimension [24]

$$M = \dim H(E) \leq \dim H(S_i) \dim H(S_f) \tag{2.5}$$

When the initial state of the environment is a mixed state

$$\rho_i(E) = \sum_{m,n=1}^{M} p_{mn} |e_m >< e_n| \tag{2.6}$$

the diagonal elements $p_{mm} \geq 0$ and

$$Tr\rho_i(E) = \sum_{m=1}^{M} p_{mm} = 1 \tag{2.7}$$

The reduced density matrix (2.2) then takes the form

$$\rho_f(S_f) = \sum_{\ell=1}^{M} \sum_{m,n=1}^{M} p_{mn} A_{\ell m} \rho_i(S_i) A_{\ell n}^{+} \tag{2.8}$$

where $A_{\ell m} =< e_\ell|U|e_m >$. In order for the reduced density matrix (2.8) to be completely positive it must acquire the form of the Kraus representation (2.3). For arbitrary states $\rho_i(E)$ this will happen if and only if the unitary evolution operator U conserves the quantum numbers of the quantum states $|e_\ell >$

$$A_{\ell m} = A_\ell \delta_{\ell m} \tag{2.9}$$

where $A_\ell =< e_\ell|U|e_\ell >$. Then (2.8) reads

$$\rho_f(S_f) = \sum_{\ell=1}^{M} p_{\ell\ell} A_\ell \rho_i(S_i) A_\ell^{+} = \sum_{\ell=1}^{M} p_{\ell\ell} \rho_f(S_f; \ell) \tag{2.10}$$

With the replacement $\sqrt{p_{\ell\ell}} A_\ell \to A_\ell$ we recover the form (2.3). We shall refer to $\rho_f(S_f; \ell)$ as Kraus density matrices. The final state $\rho_f(S_f)$ is a mixed state even when the initial state $\rho_i(S_i)$ is a pure state. For trace preserving maps the Kraus operators A_ℓ satisfy a completness relation similar to (2.4)

$$\sum_{\ell=1}^{M} p_{\ell\ell} A_\ell^{+} A_\ell = I \tag{2.11}$$

A trace preserving Kraus representation is called bistochastic if it leaves invariant the maximally mixed state of the system

$$\rho_i(S) = \frac{1}{N} \sum_{n=1}^{N} |n >< n|. \tag{2.12}$$

The Kraus representation (2.10) is a bistochastic map when the Kraus operators A_ℓ are unitary. In that case the map (2.10) is referred to as a random external field [25].

When $S_f = S_i = S$ the effect of the non-unitary interaction with the environment described by Kraus representation is a modification (dephasing) of the phases of the quantum state $\rho_i(S)$. In dissipative dephasing interactions the system S and the environment E exchange energy-momentum. There is no exchange of energy-momentum or angular momentum in pure dephasing interactions which effect only a change of phases.

A very important concept of decoherence free subspace was introduced in Ref. [26]. It describes a decoupling of a subsystem S' of the system S from the environment. In this case all Kraus operators are equal to a unitary operator $A_\ell = A$ and the Kraus representation is reduced to a unitary evolution law for that subsystem $\rho_f(S') = A\rho_i(S')A^{+}$.

III. NON-UNITARY EVOLUTION IN PARTICLE SCATTERING.

A. Evolution of produced states $\rho_f(S)$ into the observed states $\rho_f(O)$

Unitary S-matrix gives rise to a unitary evolution law

$$\rho(S) = S\rho_i S^+ \tag{3.1}$$

The initial state has a general form

$$\rho_i = \sum_{\nu,\nu'} (\rho_i)_{\nu,\nu'} |p_i,\nu;\gamma_i >< p_i,\nu';\gamma_i| \tag{3.2}$$

where p_i, ν, γ_i are the initial four-momenta, helicities and quantum numbers of the state. In the following we suppress the labels p_i and γ_i. Using a completness relation

$$\sum_f \sum_{\xi_f} \int d\Phi_f |p_f,\xi_f,\gamma_f >< p_f,\xi_f,\gamma_f| = I \tag{3.3}$$

the density matrix $\rho(S)$ has an explicit form

$$\rho(S) = \sum_f \rho_f(S) + \sum_{f'} \sum_{f''\neq f'} \rho_{f'f''}(S) \tag{3.4}$$

where

$$\begin{aligned} \rho_f(S) &= \sum_{\nu,\nu'} \sum_{\xi'_f,\xi''_f} \int d\Phi'_f d\Phi''_f |p'_f,\xi'_f,\gamma_f >< p'_f,\xi'_f,\gamma_f|S|\nu > \\ &\quad (\rho_i)_{\nu\nu'} < \nu'|S^+|p''_f,\xi''_f,\gamma_f >< p''_f,\xi''_f,\gamma_f| \\ \rho_{f'f''}(S) &= \sum_{\nu,\nu'} \sum_{\xi_{f'},\xi_{f''}} \int d\Phi_{f'} d\Phi_{f''} |p_{f'},\xi_{f'},\gamma_{f'} >< p_{f'},\xi_{f'},\gamma_{f'}|S|\nu > \\ &\quad (\rho_i)_{\nu\nu'} < \nu'|S^+|p_{f''},\xi_{f''},\gamma_{f''} >< p_{f''},\xi_{f''},\gamma_{f''}| \end{aligned} \tag{3.5}$$

The first sum in (3.3) is over all allowed final states f, the second sum is over final state helicities ξ_f and the integration is over the entire phase space of final state momenta p_f. The symbol γ_f labels the quantum numbers of the state f.

We assume that particle scattering processes are not isolated events in the Universe as implied by the unitary evolution law but behave as open quantum systems interacting with a universal quantum environment of quantum states $\rho(E)$. If the quantum environment and its interaction with particle scattering processes are to be an integral part of the Nature then they must be fully consistent with the Standard Model. This is possible if and only if this new kind of particle interaction with the quantum states $\rho(E)$ evolves the final state $\rho(S)$ produced by the S-matrix dynamics into a new observed state $\rho(O)$. The observed final state is described by the non-unitary Kraus representation

$$\rho(O) = \sum_{\ell=1}^{M} p_{\ell\ell} V_\ell \rho(S) V_\ell^+ = \sum_f \rho_f(O) + \sum_{f'} \sum_{f''\neq f'} \rho_{f'f''}(O) \tag{3.6}$$

where

$$
\begin{aligned}
\rho_f(O) &= \sum_{\ell=1}^{M} p_{\ell\ell} V_\ell \rho_f(S) V_\ell^+ && (3.7) \\
\rho_{f'f''}(O) &= \sum_{\ell=1}^{M} p_{\ell\ell} V_\ell \rho_{f'f''}(S) V_\ell^+ && (3.8)
\end{aligned}
$$

The Kraus operators V_ℓ are unitary and the state $\rho(E)$ is Poincare invariant. The consistency with the symmetries and conservation laws of the Standard Model requires that the interaction be a pure dephasing interaction that modifies only the phases of the S-matrix amplitudes in the state $\rho_f(O)$, and nothing else. There is no exchange of four-momentum and angular momentum between the environment and the produced state $\rho_f(S)$. The total four-momentum and total angular momentum are conserved by the Kraus operators in the process $|i> \to |f>$. The Kraus operators also preserve the identities of all final state particles including the four-momenta of individual particles. This means that we need to concern ourselves only with the projective measurements of the states $\rho_f(O)$ as the projections of $\rho_{f'f''}(O)$ vanish.

B. Experimental signature of the dephasing interaction

How do we observe the change in the in the phases of the S-matrix amplitudes in the observed amplitudes from the measurements of the density matrix $\rho_f(O)$? To answer this question consider three complex functions A, B, C. Their relative phases $\Phi(AB^*) = \Phi(A) - \Phi(B)$ satisfy the phase condition

$$
\Phi(AB^*) = \Phi(AC^*) + \Phi(CB^*) \tag{3.9}
$$

and the equivalent cosine condition

$$
\cos^2 \Phi(AB^*) + \cos^2 \Phi(AC^*) + \cos^2 \Phi(CB^*) - 2\cos\Phi(AB^*)\cos\Phi(AC^*)\cos\Phi(CB^*) = 1 \tag{3.10}
$$

In general the measurement determines the moduli $|A|^2$ of the amplitudes and their correlations $\cos\Phi(AB)$. If the correlations violate the conditions (3.10) on the cosines then the observed amplitudes are not complex valued functions and thus cannot be S-matrix amplitudes. The non-unitary correlations imply that a pure intial state evolves into a mixed final state. The scattering process therefore interacts with the quantum environment.

If all cosine conditions are satisfied then the measured amplitudes are complex valued functions. Can these amplitudes be identified with the S-matrix amplitudes? The answer is not necessarily.

Suppose that the dephasing interaction with the environment simply shifts the phases of the S-matrix amplitudes A from $\Phi(A)$ to $\Psi(A) = \Phi(A) + \delta(A)$. The observed correlations will be $\cos(\Psi(A) - \Psi(B))$ and describe complex valued functions. When the unitary evolution law imposes specific constraints on the relative phases $\Phi(A) - \Phi(B)$ from the condition that pure states evolve into pure states, we will be able to discern from the observed relative phases that the scattering process interacts with an environment. Only when no such constraints exist, and there is no violation of the cosine conditions on the correlations, we can identify the observed amplitudes with the S-matrix amplitudes. The scattering process in this case does not interact with the quantum environment.

It is our conjecture that when the unitary evolution from pure states to pure states imposes some special constraints on the relative phases of S-matrix amplitudes then these unitary phases are always modified in the observed amplitudes. In other words, in such cases the particle scattering

or decay process $|i>\to|f>$ always interacts with the environment. When the unitary evolution law constraints on the amplitudes are all identities and thus impose no special unitary phases, then any observed phases can be identified with the phases of the S-matrix amplitudes and we say such process $i>\to|f>$ does not interact with the environment.

S-matrix evolution law evolves pure states into pure states for free particles and two-body scattering processes like of scalar-scalar scattering $0+0\to 0+0$ or meson-nucleon scattering $0+\frac{1}{2}\to 0+\frac{1}{2}$ without imposing any constraints on the phases of the amplitudes. These processes thus do not interact with the environment.

It is our conjecture that all two-body processes do not interact with the environment. The reason for this conjecture is the fact that two-body processes produce no superpositions of diparticle states with variable mass and spin [27–29] in the final state which, as we shall see below, are directly involved in the dephasing interaction. If two-body reactions do not interact with quantum environment, then their amplitudes must be analytical complex functions which implies a Froissart bound on the total cross-sections [27]. Froissart bound has been recently confirmed in pp scattering up to 7000 GeV at LHC [30, 31].

C. The dimension M of the Hilbert space $H(E)$

We can now turn to the question of the dimension M of the Hilbert space $H(E)$. Recall that $M=\dim H(E)\leq \dim H(S_i)\dim H(S_f)$ in a non-unitary process $\rho(S_i)\to\rho(S_f)$. In (3.7) $\rho(S_i)=\rho_f(S)$ and $\rho(S_f)=\rho_f(O)$. The relevant Hilbert space $H(O)=H(S)$ is the space of spin states for all final state particles forming the states $\rho_f(S)$ and $\rho_f(O)$. The dimension M must be common to all processes.

The simplest scattering process that involves interaction with the environment is $\pi N\to\pi\pi N$. Its simplest spin state describes the S-wave process $\pi N\to 0^{++}N$. The evolution of $0^{++}N\to 0^{++}N$ due to its interaction with the environment limits the dimension M to $1\leq M\leq(2\frac{1}{2}+1)(2\frac{1}{2}+1)=4$. The case $M=1$ is excluded by the by the S-and P-wave amplitude analyses of the CERN data [17, 20, 21] and by the consistency of the pure dephasing interaction with the S-matrix (Section V.C). In a related study [21] of Kraus representation in $\pi^-p\to\pi^-\pi^+n$ we show that the dimension $M=4$.

IV. DEPHASING INTERACTION AND THE SPIN MIXING MECHANISM IN $\pi N\to\pi\pi N$.

A. S-matrix final state $\rho_f(S)$ in $\pi N\to\pi\pi N$

The pion creation process $\pi_a N_b\to\pi_1\pi_2 N_d$ is the very simplest particle scattering process where we can observe non-unitary evolution and the spin mixing interaction. With four-momenta $p_a+p_b\to p_1+p_2+p_d$ the initial and final particle states of the process are described by state vectors $|p_ap_b;0\nu>$ and $|p_cp_d;\theta\phi;\chi>$ where $p_c=p_1+p_2$ and ν and χ are the helicities of the target and recoil nucleon, respectively. The direction of π_1 in dipion center-of-mass system is descibed by θ,ϕ. Target nucleon spin density matrix is $\rho_b(\vec{P})$ where $\vec{P}$ is target polarization. The produced final state is given by the S-matrix unitary evolution law

$$\rho_f(S)=\int d\Phi_3 d\Phi_3'\sum_{\xi\xi'}R(S,\vec{P})_{\xi\xi'}|p_cp_d;\theta\phi,\xi><p_c'p_d';\theta'\phi',\xi'| \tag{4.1}$$

where

$$R(S,\vec{P})_{\xi\xi'} = \sum_{\nu\nu'} S_{\xi,0\nu}(p_c p_d,\theta\phi)\rho_b(\vec{P})_{\nu\nu'} S^*_{\xi',0\nu'}(p'_c p'_d,\theta'\phi') \tag{4.2}$$

and where we have used the phase space relation [27]

$$d\Phi_3 = \frac{d^3\vec{p}_1}{2E_1}\frac{d^3\vec{p}_2}{2E_2}\frac{d^3\vec{p}_d}{2E_d} = \frac{q}{4m}d^4p_c d\Omega \frac{d^3\vec{p}_d}{2E_d} = d\overline{\Phi}_3 d\Omega \tag{4.3}$$

in the completness relation. The angular S-matrix amplitudes are matrix elements

$$\begin{aligned} S_{\xi,0\nu}(p_c p_d,\theta\phi) &= < p_c p_d;\theta\phi,\xi|S|p_a p_b,0\nu > \\ S^*_{\xi',0\nu'}(p'_c p'_d,\theta'\phi') &= < p_a p_b,0\nu'|S^+|p'_c p'_d;\theta'\phi',\xi' > \end{aligned} \tag{4.4}$$

Prior to the interaction of the produced final state with the quantum environment there is no measurement and thus no measurement projection of this state to $\rho_f(p_c p_d,\theta\phi;\vec{P}))$. The angular state $|p_c p_d;\theta\phi,\xi >$ can be expanded in terms of spherical harmonics [20]

$$|p_c p_d;\theta\phi,\xi >= \sum_{K=0}^{\infty}\sum_{\mu=-K}^{K} Y^{K*}_{\mu}(\theta,\phi)|p_c p_d;K\mu,\xi > \tag{4.5}$$

where K and μ are the dipion spin and helicity, respectively. Integrating over $d\Omega$ and $d\Omega'$ and using the relation

$$\int d\Omega Y^K_\mu(\Omega) Y^{K'*}_{\mu'}(\Omega) = \delta_{KK'}\delta_{\mu\mu'} \tag{4.6}$$

we find

$$\rho_f(S) = \int d\overline{\Phi}_3 d\overline{\Phi'}_3 \sum_{K\mu,\xi}\sum_{K'\mu',\xi'} R(S,\vec{P})^{K\frac{1}{2},K'\frac{1}{2}}_{\mu\xi,\mu'\xi'}|p_c p_d;K\mu,\xi >< p'_c p'_d;;K'\mu',\xi'| \tag{4.7}$$

where the joint spin density matrix elements

$$R(S,\vec{P})^{K\frac{1}{2},K'\frac{1}{2}}_{\mu\xi,\mu'\xi'}(p_c p_d,p'_c p'_d) = \sum_{\nu,\nu'} S^K_{\mu\xi,0\nu}(p_c p_d)\rho_b(\vec{P})_{\nu\nu'} S^{K'*}_{\mu'\xi',0\nu'}(p'_c p'_d) \tag{4.8}$$

The partial wave S-matrix amplitudes are matrix elements

$$\begin{aligned} S^K_{\mu\xi,0\nu}(p_c p_d) &= < p_c p_d;K\mu,\xi|S|p_a p_b,0\nu > \\ S^{K'*}_{\mu'\xi',0\nu'}(p'_c p'_d) &= < p_a p_b,0\nu'|S^+|p'_c p'_d;K'\mu',\xi' > \end{aligned} \tag{4.9}$$

In general, the final state density matrix (4.7) is a mixed state of entangled dipion and recoil nucleon helicity helicity states $|K\mu >$ and $|\xi >$ with different dipion spins K. Equation (4.7) can be written in a physically important form

$$\rho_f(S) = \int d\overline{\Phi}_3 d\overline{\Phi'}_3 \sum_{\nu\nu'} \rho_i(\vec{P})_{\nu\nu'}|\Psi(p_c p_d,0\nu) >< \Psi(p'_c p'_d,0\nu')| \tag{4.10}$$

where $|\Psi(p_c p_d,0\nu) >$ and $< \Psi(p'_c p'_d,0\nu')|$ are superpositions of partial waves $|p_c p_d;K\mu,\xi >$ given by

$$\begin{aligned} |\Psi(p_c p_d,0\nu) > &= \sum_{K\mu,\xi} S^K_{\mu\xi,0\nu}(p_c p_d)|p_c p_d;K\mu,\xi > \\ < \Psi(p'_c p'_d,0\nu')| &= \sum_{K'\mu',\xi'} < p'_c p'_d;K'\mu',\xi'|S^{K'*}_{\mu'\xi',0\nu'}(p'_c p'_d) \end{aligned} \tag{4.11}$$

B. Spin mixing mechanism

We now use the Kraus representation (3.7) to determine the nature and the form of the pure dephasing interaction with the quantum environment in $\pi N \to \pi\pi N$ processes. The unitary Kraus operators V_ℓ act on the superposions (4.11) of the partial waves $|p_c p_d; K\mu, \xi >$ by rotating the dipion spin states $|K\mu, \xi >$. Applying the completness relation (3.3) from left to $V_\ell |p_c p_d; K\mu, \xi >$ we find

$$\begin{aligned} V_\ell |p_c p_d; K\mu, \xi > &= \sum_f \sum_{\xi_f} \int d\Phi_f |p_f, \xi_f, \gamma_f >< p_f, \xi_f, \gamma_f | V_\ell | p_c p_d; K\mu, \xi > \\ &= \sum_\chi \int d\Omega |p_c p_d; \theta\phi, \chi >< p_c p_d; \theta\phi, \chi | V_\ell | p_c p_d; K\mu, \xi > \end{aligned} \tag{4.12}$$

since the Kraus operators preserve the four-momenta and the identity of the final state particles. Using the identity

$$\sum_\chi \int d\Omega |\theta\phi, \chi >< \theta\phi, \chi| = \sum_{J\lambda,\chi} |J\lambda, \chi >< J\lambda, \chi| = I \tag{4.13}$$

the equation (4.12) then acquires the form

$$V_\ell |p_c p_d; K\mu, \xi >= \sum_{J\lambda,\chi} < p_c p_d; J\lambda, \chi | V_\ell | p_c p_d; K\mu, \xi > |p_c p_d; J\lambda, \chi > \tag{4.14}$$

This relation shows the interaction of the produced state $\rho_f(S)$ with the environment is a dipion spin mixing interaction. Its effect is to produce a new superposition of partial waves $|p_c p_d; J\lambda, \chi >$

$$|\Psi_\ell(p_c p_d, 0\nu) >= V_\ell |\Psi(p_c p_d, 0\nu) >= \sum_{J\lambda,\chi} A^J_{\lambda\chi,0\nu}(p_c p_d; \ell) |p_c p_d; J\lambda, \chi > \tag{4.15}$$

Here $A^J_{\lambda\chi,0\nu}(p_c p_d; \ell)$ are Kraus partial wave amplitudes

$$A^J_{\lambda\chi,0\nu}(p_c p_d; \ell) = \sum_{K,\mu} \sum_\xi < p_c p_d; J\lambda, \chi | V_\ell | p_c p_d; K\mu, \xi >< p_c p_d; K\mu, \xi | S | p_a p_b, 0\nu > \tag{4.16}$$

which explicitely show the mixing of S-matrix partial wave amplitudes with different dipion spins K and helicities μ. We refer to the relation (4.16) as spin mixing mechanism. Applying the relation (4.13) to the combination

$$\sum_{J\lambda,\chi} |p_c p_d; J\lambda, \chi >< p_c p_d; J\lambda, \chi | V_\ell | p_c p_d; K\mu, \xi > \tag{4.17}$$

in (4.15) we transform $|\Psi_\ell(p_c p_d, 0\nu) >$ into an angular form

$$|\Psi_\ell(p_c p_d, 0\nu) >= \int d\Omega |p_c p_d; \theta\phi, \chi > A_{\chi,0\nu}(p_c p_d, \theta\phi; \ell) \tag{4.18}$$

where the Kraus angular amplitudes

$$A_{\chi,0\nu}(p_c p_d, \theta\phi; \ell) = \sum_{K\mu,\xi} < p_c p_d; \theta\phi, \chi | V_\ell | p_c p_d; K\mu, \xi >< p_c p_d; K\mu, \xi | S | p_a p_b, 0\nu > \tag{4.19}$$

Similarly we find for $<\Psi_\ell(p'_c p'_d, 0\nu')| = <\Psi(p'_c p'_d, 0\nu')|V_\ell^+$

$$<\Psi_\ell(p'_c p'_d, 0\nu')| = \sum_{J'\lambda',\chi'} A^{J'*}_{\lambda'\chi',0\nu'}(p'_c p'_d;\ell) < p'_c p'_d; J'\lambda', \chi'| \tag{4.20}$$

$$= \int d\Omega' A^{*}_{\chi',0\nu'}(p'_c p'_d, \theta'\phi';\ell) < p'_c p'_d; \theta'\phi', \chi'|$$

where the Kraus amplitudes

$$A^{J'*}_{\lambda'\chi',0\nu'}(p'_c p'_d;\ell) = \sum_{K'\mu',\xi'} < p_a p_b, 0\nu'|S^+|p'_c p'_d, K'\mu', \xi' >< p'_c p'_d; K'\mu', \xi'|V_\ell^+|p'_c p'_d; J'\lambda', \chi' > \tag{4.21}$$

$$A^{*}_{\chi',0\nu'}(p'_c p'_d, \theta'\phi';\ell) = \sum_{K'\mu',\xi'} < p_a p_b, 0\nu'|S^+|p'_c p'_d, K'\mu', \xi' >< p'_c p'_d; K'\mu', \xi'|V_\ell^+|p'_c p'_d; \theta'\phi', \chi' >$$

The Kraus representation (3.7) then reads

$$\rho_f(O) = \sum_{\ell=1}^{4} p_{\ell\ell}\rho_f(\vec{P};\ell\ell) \tag{4.22}$$

where

$$\begin{aligned}\rho_f(\vec{P};\ell\ell) \; = \; & \int d\Phi_3 d\Phi'_3 \sum_{\chi\chi'}\sum_{\nu\nu'} |p_c p_d; \theta\phi, \chi >< p'_c p'_d; \theta'\phi', \chi'| \\ & A_{\chi,0\nu}(p_c p_d, \theta\phi;\ell)\rho_b(\vec{P})_{\nu\nu'} A^{*}_{\chi',0\nu'}(p'_c p'_d, \theta'\phi';\ell)\end{aligned} \tag{4.23}$$

The equations (4.23) have a formal resemblance to the equation (4.1) for the S-matrix final state $\rho_f(S)$. It is evident from the definitions of Kraus partial wave and angular amplitudes (4.16), (4.19) and (4.21) that these amplitudes cannot be written as matrix elements of a unitary operator in contrast to the S-matrix partial wave amplitudes if not all spin states $|K\mu, \xi>$ are allowed and (4.13) does not apply. In that case the Kraus partial wave amplitudes shall violate the unitarity conditions on relative phases of the S-matrix partial wave amplitudes [20]. As a result they are complex valued functions with non-unitary phases. Spins K are resticted by the conditions (5.6).

C. Projective measurement and Kraus helicity amplitudes

Following a projective measurement of the final state $\rho_f(O)$ into a state with definite four-momenta p_c, p_d and the direction $\theta\phi$ of the pion π_1, the Kraus representation (4.22) takes the form

$$\rho_f(p_c p_d, \theta\phi, \vec{P}) = \sum_{\ell=1}^{4} p_{\ell\ell}\rho_f(p_c p_d, \theta\phi, \vec{P};\ell\ell) \tag{4.24}$$

Suppressing the four-momenta in the initial and final state vectors $|0\nu>$ and $|\theta\phi, \chi>$, the Kraus density matrices read

$$\rho_f(\theta\phi, \vec{P};\ell\ell)_{\chi\chi'} = \sum_{\nu\nu'} A_{\chi,0\nu}(\theta\phi;\ell)\rho_b(\vec{P})_{\nu\nu'} A_{\chi',0\nu'}(\theta\phi;\ell) \tag{4.25}$$

With $S = I + i(2\pi)^4\delta(P_f - P_i)T$ we have from (4.13)

$$A_{\chi,0\nu}(\theta\phi;\ell) = i(2\pi)^4\delta(P_f - P_i)T_{\chi,0\nu}(\theta\phi;\ell) \tag{4.26}$$

where $T_{\chi,0\nu}(\theta\phi;\ell)$ are the new Kraus angular amplitudes. Following formally the steps employed in the case of the S-matrix spin formalism [20] we can redefine the Kraus density matrices (4.25) in terms of new Kraus helicity amplitudes

$$H_{\chi,0\nu}(\theta\phi;\ell) = K(s,m^2)T_{\chi,0\nu}(\theta\phi;\ell) \tag{4.27}$$

The normalization factor

$$K(s,m^2) = \sqrt{\frac{q(m^2)G(s)}{Flux(s)}} \tag{4.28}$$

where $m^2 = p_c^2$ is the dipion mass, s is the center-of-mass energy squared, $q(m^2)$ is pion momentum in dipion center-of-mass, $G(s)$ and $Flux(s)$ are the energy dependent part of the phase space and flux, respectively [13]. The redefined matrices (4.25) then read

$$\rho_f(\theta\phi,\vec{P};\ell\ell)_{\chi\chi'} = \sum_{\nu\nu'} H_{\chi,0\nu}(\theta\phi;\ell)\rho_b(\vec{P})_{\nu\nu'}H^*_{\chi',0\nu'}(\theta\phi;\ell) \tag{4.29}$$

Angular expansion of Kraus helicity amplitudes

$$H_{\chi,0\nu}(\theta\phi;\ell) = \sum_{J=0}^{\infty}\sum_{\lambda=-J}^{J} H^J_{\lambda\chi,0\nu}(\ell)Y^J_\lambda(\theta,\phi) \tag{4.30}$$

defines Kraus partial wave helicity amplitudes with a definite dipion spin and helicity

$$H^J_{\lambda\chi,0\nu}(\ell) = K(s,m^2)T^J_{\lambda\chi,0\nu}(\ell) \tag{4.31}$$

So far we have suppressed isospin labels of the produced pions. In strong interactions pions are considered as identical particles. The ensuing Bose-Einstein statistics for two-pion states requires that $I + J = even$ where I is the total dipion isospin [27]. Only $J = even$ is allowed for identical pions. For different pions S-matrix partial waves with $I + J = odd$ vanish while for identical pions amplitudes with $J = odd$ vanish. Labeling the isospin states I_J, the spin mixing mechanism for Kraus partial wave helicity amplitudes reads

$$H^J_{\lambda\chi,0\nu}(\ell) = \sum_{K,\mu}\sum_{\xi} < p_cp_d; J\lambda I_J,\chi|V_\ell|p_cp_d; K\mu I_K,\xi > H^K_{\mu\xi,0\nu}(S) \tag{4.32}$$

where $H^K_{\mu\xi,0\nu}(S)$ are the redefined S-matrix partial wave helicity amplitudes [20]. The isospin of the Kraus partial wave helicity amplitudes is defined to be I_J although for different pions these amplitudes mix isospin as the result of spin mixing. Kraus partial wave amplitudes with $J = odd$ still vanish for identical pions due to the Bose-Einstein statistics for pion spin.

V. CONSISTENCY OF THE DEPHASING INTERACTION WITH THE STANDARD MODEL.

A. Lorentz symmetry of the Kraus operators

We refer to the matrix elements $< p_cp_d; J\lambda I_J,\chi|V_\ell|p_cp_d; K\mu I_K,\xi >$ as dephasing amplitudes. They describe a forward two-body scattering of dipion states and recoil nucleons

$$|p_c, K\mu I_K > + |p_d, \frac{1}{2}\mu > \rightarrow |p_c, J\lambda I_J > + |p_d, \frac{1}{2}\chi > \tag{5.1}$$

with the Kraus operators V_ℓ taking on the role of scattering operators. As in the case of S-matrix scattering, there is a priori no restriction on the final spin J and helicity λ.

To be consistent with the Standard Model, the dephasing interaction must be Lorentz invariant. That means that all Kraus operators must be Lorentz invariant. The rescattering process of the dipions with the recoil nucleon must thus conserve the total four-momentum and total angular momentum.

The dephasing amplitudes obviously conserve the total four-momentum. They must depend on the Lorentz invariants of the four-momenta p_c and p_d, i.e. the total energy s and dipion mass m^2. To examine their conservation of the total angular momentum we go into the center-of-mass system in the initial and final states of the process (4.33) where the dipion has 3-momentum q and direction θ_c, ϕ_c. Then [27]

$$
\begin{aligned}
|p_c p_d, K\mu I_K, \xi > &= \sqrt{\frac{4s}{q}} \sum_{j_i m_i} \sqrt{\frac{2j_i+1}{4\pi}} \mathcal{D}^{j_i}_{m_i \mu'}(\phi_c, \theta_c, -\phi_c) |q, j_i m_i, K\mu I_K, \xi > \\
< p_c p_d, J\lambda I_J, \chi| &= \sqrt{\frac{4s}{q}} \sum_{j_f m_f} \sqrt{\frac{2j_f+1}{4\pi}} \mathcal{D}^{j_f *}_{m_f \lambda'}(\phi_c, \theta_c, -\phi_c) < q, j_f m_f, J\lambda I_J, \chi|
\end{aligned}
\tag{5.2}
$$

where $\mu' = \mu - \xi$ and $\lambda' = \lambda - \chi$. The expansion of the dephasing amplitudes defines partial wave amplitudes in $< q, j_f m_f, J\lambda I_J, \chi|V_\ell|q, j_i m_i, K\mu I_K, \xi >$ which conserve the total angular momentum $j_i = j_f$, $m_i = m_f$ and which are independent of m_i due to rotational invariance. Then

$$
< p_c p_d; J\lambda I_J, \chi|V_\ell|p_c p_d; K\mu I_K, \xi >= \delta_{\lambda'\mu'} < q; J\lambda I_J, \chi|V_\ell|q; K\mu I_K, \xi > \tag{5.3}
$$

where

$$
< q; J\lambda I_J, \chi|V_\ell|q; K\mu I_K, \xi >= \frac{4s}{q} \sum_{j_i} \frac{2j_i+1}{4\pi} < q; j_i, J\lambda I_J, \chi|V_\ell|q; j_i, K\mu I_K, \xi > \tag{5.4}
$$

We see that the dephasing amplitudes are non-zero only for

$$
\lambda - \chi = \mu - \xi \tag{5.5}
$$

We can write the conservation of total angular momentum $j_f = j_i$ in the form $L_i + S_i = L_f + S_f$ where L_i and L_f are the initial and final orbital momenta while S_i and S_f are the initial and final total spins. Since the four-momenta of the particles in the initial and final states are equal we have $L_i = L_f$. For total spins we have $S_i = K \pm \frac{1}{2}$ and $S_f = J \pm \frac{1}{2}$. Then $S_i = S_f$ implies that the only possible values of the spin K are

$$
K = J-1, J, J+1 \tag{5.6}
$$

B. Consistency of Kraus helicity amplitudes

We require that the Kraus helicity amplitudes (4.32) be consistent with the Standard Model, in particular with the conservation of P-parity, total four-momentum and total angular momentum, total isospin, and with the conservation of probability imposed by the S-matrix. These requirements will impose constraints on the matrix elements of the Kraus operators that ensure the dipion spin mixing interaction is a pure dephasing interaction.

To ensure that all observers can communicate the same information about the production process and its interaction with the quantum environment the Kraus partial wave helicity amplitudes

(4.32) must transform as two-body helicity S-matrix amplitudes [27] under Lorentz transformations. This in turn requires that the spin mixing mechanism is Lorentz invariant.

Let Λ be a homogeneous Lorentz transformation from a frame Σ to a frame Σ'. The transformation law of the helicity amplitudes is obtained from

$$U(\Lambda)|p,\mu>=\sum_{\mu'\mu}\mathcal{D}^s_{\mu'\mu}(R)|p',\mu'> \tag{5.7}$$

In (5.7) $p'=\Lambda p$ and $\mathcal{D}^s_{\mu'\mu}(R)$ is the matrix representing Wigner rotation $R(\Lambda,p)=\Lambda_p^{-1}\Lambda\Lambda_p$. Here $\Lambda_p=R(\theta,\phi)Z_p$ where Z_p is a boost Lorentz transformation along the z-axis followed by a rotation through Euler angles θ,ϕ.

With Lorentz symmetry of the S-matrix and Kraus operators, the transformation laws for the S-matrix and dephasing amplitudes read [27]

$$< p_cp_d;K\mu,\xi|T|p_ap_b;0\nu>= \tag{5.8}$$

$$\sum_{\mu'\xi'\nu'}(-1)^{\nu'-\nu+\xi'-\xi}\mathcal{D}^{K*}_{\mu'\mu}(R_c)\mathcal{D}^{\frac{1}{2}*}_{\xi'\xi}(R_d)< p'_cp'_d;K\mu',\xi'|T|p'_ap'_b;0\nu'>\mathcal{D}^{\frac{1}{2}}_{\nu'\nu}(R_b)$$

$$< p_cp_d;J\lambda,\chi|V_\ell|p_cp_d;K\mu,\xi>= \tag{5.9}$$

$$\sum_{\lambda'\chi'}\sum_{\mu''\xi''}(-1)^{\xi''-\xi+\chi'-\chi}\mathcal{D}^{J*}_{\lambda'\lambda}(R_c)\mathcal{D}^{\frac{1}{2}*}_{\chi'\chi}(R_d)< p'_cp'_d;J\lambda',\chi'|V_\ell|p'_cp'_d;K\mu'',\xi''>\mathcal{D}^{K}_{\mu''\mu}(R_c)\mathcal{D}^{\frac{1}{2}}_{\xi''\xi}(R_d)$$

where we suppressed the isospin labels for the sake of brevity. Combining these results in the expression (4.32) for the Kraus helicity amplitudes and using

$$\sum_{\mu\xi}\mathcal{D}^{K}_{\mu''\mu}(R_c)\mathcal{D}^{\frac{1}{2}}_{\xi''\xi}(R_d)\mathcal{D}^{K*}_{\mu'\mu}(R_c)\mathcal{D}^{\frac{1}{2}*}_{\xi'\xi}(R_d)=\delta_{\mu''\mu'}\delta_{\xi''\xi'} \tag{5.10}$$

we obtain

$$H^J_{\lambda\chi,0\nu}(p_cp_d,\ell)= \tag{5.11}$$

$$\sum_{\lambda'\chi'}\sum_{\nu'}(-1)^{\chi'-\chi+\nu'-\nu}\mathcal{D}^{J*}_{\lambda'\lambda}(R_c)\mathcal{D}^{\frac{1}{2}*}_{\chi'\chi}(R_d)H^J_{\lambda'\chi',0\nu'}(p'_cp'_d,\ell)\mathcal{D}^{\frac{1}{2}}_{\nu'\nu}(R_b)$$

This is the required transformation law for the Kraus helicity amplitudes which also shows that the spin mixing mechanism is Lorentz invariant.

The requirement of P-parity conservation implies that the Kraus helicity amplitudes satisfy the same parity relations as the corresponding S-matrix helisity amplitudes, namely [20]

$$H^J_{-\lambda-\chi,0-\nu}(\ell)=(-1)^{\lambda+\chi+\nu}H^J_{\lambda\chi,0\nu}(\ell) \tag{5.12}$$

The relation (4.32) then requires that

$$< p_cp_d;J-\lambda I_J,-\chi|V_\ell|p_cp_d;K-\mu I_K,-\xi>= \tag{5.13}$$

$$(-1)^{\lambda+\chi-\mu-\xi}< p_cp_d;J\lambda I_J,\chi|V_\ell|p_cp_d;K\mu I_K,\xi>$$

Since the dephasing interaction preserves the four-momenta of the all final state particles, the total four-momentum is obviously conserved by Kraus helicity amplitudes. To show they conserve the total angular momentum we consider the S-matrix and Kraus amplitudes as functions of the c.m.s. scattering angle θ_c, ϕ_c. Then the partial wave expansion of the final two-body state in S-matrix and Kraus amplitudes reads

$$H^K_{\mu\xi,0\nu}(S,\theta_c\phi_c) = \sqrt{\frac{4s}{q}} \sum_{J_T,M_T} \sqrt{\frac{2J_T+1}{4\pi}} \mathcal{D}^{J_T}_{M_T M'}(\phi_c,\theta_c,-\phi_c) H^K_{\mu\xi,0\nu}(S,J_T M_T) \tag{5.14}$$

$$H^J_{\lambda\chi,0\nu}(\ell,\theta_c\phi_c) = \sqrt{\frac{4s}{q}} \sum_{J_T,M_T} \sqrt{\frac{2J_T+1}{4\pi}} \mathcal{D}^{J_T}_{M_T M''}(\phi_c,\theta_c,-\phi_c) H^J_{\lambda\chi,0\nu}(\ell,J_T M_T) \tag{5.15}$$

where J_T and M_T are the total angular mometum and its z-axis component, and $M' = \mu - \xi$, $M'' = \lambda - \chi$. Assuming $M' = M''$, or $\lambda - \chi = \mu - \xi$, we can write

$$H^J_{\lambda\chi,0\nu}(\ell,J_T M_T) = \sum_{K\mu,\xi} < p_c p_d; J\lambda I_J,\chi|V_\ell|p_c p_d; K\mu I_K,\xi > H^K_{\mu\xi,0\nu}(S,J_T M_T) \tag{5.16}$$

which ensures the conservation of the total angular momentum by the Kraus helicity amplitudes. Note that the equality $M' = M''$ follows from the equation (5.5), i.e. from the Lorentz symmetry of the Kraus operators.

The conservation of the total isospin in Kraus helicity amplitudes follows from the fact that the isospin identities of the final state particles are preserved by the dephasing interaction. Consider $\pi^- p \to \pi^-\pi^+ n$ process. For $K = odd$ isospin state $|I_K >= |1,0>$ while for $K = even$ the isospin state $|I_K >= a|0,0> + b|2,0>$ is a combination of states $|0,0>$ and $|2,0>$. Since the total initial isospin is conserved by the S-matrix amplitudes in both cases, then it must be conserved by the dephasing amplitudes with the states $|I_J >= |1,0>$ and $|I_J >= a|0,0> + b|2,0>$ as well. This propagation of total isospin in the spin mechanism ensures isospin conservation by Kraus helicity amplitudes.

Unitarity of the Kraus operators requires

$$\sum_{K\mu,\xi} < p_c p_d; J\lambda I_J,\chi|V_\ell|p_c p_d; K\mu I_K,\xi >< p_c p_d; J'\lambda' I_{J'},\chi'|V_\ell|p_c p_d; K\mu I_K,\xi >^* = \delta_{JJ'}\delta_{\lambda\lambda'}\delta_{\chi\chi'} \tag{5.17}$$

The conservation of the total angular momentum constraint (5.5) requires that $\lambda - \chi = \lambda' - \chi' = \mu - \xi$ for arbitrary $\lambda, \chi, \lambda', \chi'$. For $\lambda - \chi \neq \lambda' - \chi'$ the unitarity relation is a null identity.

C. Consistency constraint $M = \dim H(E) > 1$

We shall use Coleman-Mandula No-Go Theorem [32] to show that the spin mixing mechanism is consistent with the S-matrix scattering dynamics provided the interaction with the environment is non-unitary. A modern proof of the Theorem is given by Weinberg in Ref. [33].

Assume that the generators P_μ and $M_{\mu\nu}$ of the Poincare group commute with the S-matrix. Assume that internal symmetry generators T^a (charges) commute with the S-matrix and form an algebra given by commutation relations

$$[T^a, T^b] = i f^{abc} T^c \tag{5.18}$$

Coleman-Mandula No-Go Theorem asserts that there is no physically meaningful S-matrix theory where the charges T^a do NOT commute with the generators of the Poincare group. That

means there is no mixing of space-time transformations and internal symmetry transformations. Supersymmetry evades this No-Go Theorem by replacing the commutation relations (4.12) with anticommutation relations [33].

The key assumption in the proof of the Coleman-Mandula Theorem is the assumption that a two-body scattering always exists in the S-matrix theory as opposed to a situation where only a forward scattering is possible. Theories that allow only forward scattering evade the No-Go Theorem. The matrix elements of the operators V_k describing the dephasing interaction of the "two-body" spin states $|p_c p_d; J\lambda, \chi>$ all describe two-body forward scattering of these states. Do they evade the No-Go Theorem?

To answer this question let us first assume that there is only one operator V mixing the spins. V cannot be the S-matrix for S-matrix allows the scattering of the spin states $|p_c p_d; J\lambda, \chi>$ and would automatically generate such scattering. Thus V is a genuine "No-scattering" operator. Note that the appearance of different multiparticle spin states $|p_c p_d; J\lambda, \chi>$ and $|p_c p_d; K\mu, \xi>$ in the initial and final states of V does not signify a violation of Lorentz symmetry since the forward scattering of the "two-particle" states is allowed. Thus Lorentz symmetry is conserved and the Coleman-Mandula Theorem is evaded. Then internal charges such as isospin need not to commute with the generators of the Poincare group and in fact can carry a Lorentz index. This is in a complete contradiction with the S-matrix theory including supersymmetry. To resolve this contradiction we must "evade the evasion" by assuming that there is more than one spin mixing "No-scattering" operator, i.e. $V_k, k = 1, M > 1$. But that means that the dephasing spin mixing interaction with the environmeni must be non-unitary in order to be consistent with S-matrix dynamics.

VI. SELF-CONSISTENT FORM OF THE SPIN MIXING MECHANISM.

A. Kraus transversity amplitudes

The measurements of $\pi N \to \pi\pi N$ processes on polarized targets are best analyzed in terms of transversity amplitudes with definite t-channel naturality. To define Kraus transversity amplitudes we first must define Kraus helicity amplitudes with definite t-channel naturality [20]. The unnatural exchange amplitudes are defined by

$$U^J_{\lambda+,0\pm}(\ell) = \frac{1}{\sqrt{2}}(H^J_{\lambda+,0\pm}(\ell) + (-1)^\lambda H^J_{-\lambda+,0\pm}(\ell)) \tag{6.1}$$

while the natural exchange amplitudes are given by

$$N^J_{\lambda+,0\pm}(\ell) = \frac{1}{\sqrt{2}}(H^J_{\lambda+,0\pm}(\ell) - (-1)^\lambda H^J_{-\lambda+,0\pm}(\ell)) \tag{6.2}$$

where $\chi = +$ and $\nu = \pm$ are nucleon helicities. Both amplitudes satisfy the same parity relations (5.12) as the helicity amplitudes. However to express $U^J_{\lambda\chi,0\nu}(\ell)$ and $N^J_{\lambda\chi,0\nu}(\ell)$ in terms of the S-matrix amplitudes with definite naturality we need another constraint

$$< p_c p_d; J\lambda I_J, -\chi|V_\ell|p_c p_d; K\mu I_K, -\xi >= (-1)^{\chi-\xi} < p_c p_d; J\lambda I_J, \chi|V_\ell|p_c p_d; K\mu I_K, \xi > \tag{6.3}$$

which from the P-parity relations (5.13) implies

$$< p_c p_d; J-\lambda I_J, \chi|V_\ell|p_c p_d; K-\mu I_K, \xi >= (-1)^{\lambda-\mu} < p_c p_d; J\lambda I_J, \chi|V_\ell|p_c p_d; K\mu I_K, \xi > \tag{6.4}$$

With the resulting expressions we calculate Kraus unnatural and natural exchange transversity amplitudes defined by [20]

$$
\begin{aligned}
U^J_{\lambda u}(\ell) &= \frac{1}{\sqrt{2}}(U^J_{\lambda+,0+}(\ell) + iU^J_{\lambda+,0-}(\ell)) \\
U^J_{\lambda d}(\ell) &= \frac{1}{\sqrt{2}}(U^J_{\lambda+,0+}(\ell) - iU^J_{\lambda+,0-}(\ell))
\end{aligned} \tag{6.5}
$$

for unnatural exchange amplitudes and with a similar definition

$$
\begin{aligned}
N^J_{\lambda u}(\ell) &= \frac{1}{\sqrt{2}}(N^J_{\lambda+,0+}(\ell) + iN^J_{\lambda+,0-}(\ell)) \\
N^J_{\lambda d}(\ell) &= \frac{1}{\sqrt{2}}(N^J_{\lambda+,0+}(\ell) - iN^J_{\lambda+,0-}(\ell))
\end{aligned} \tag{6.6}
$$

for the natural exchange amplitudes. In (6.5) and (6.6) $\tau = u, d$ is the target nucleon transversity. The Kraus transversity amplitudes satisfy parity relations

$$
\begin{aligned}
U^J_{-\lambda\tau}(\ell) &= +(-1)^\lambda U^J_{\lambda\tau}(\ell) \\
N^J_{-\lambda\tau}(\ell) &= -(-1)^\lambda N^J_{\lambda\tau}(\ell)
\end{aligned} \tag{6.7}
$$

For $\lambda = 0$ note that $N^J_{0+,0\pm}(\ell) = 0$ so that $N^J_{0\tau}(\ell) = 0$.

B. Self-consistency constraints on the spin mixing mechanism

Starting with (4.32) and using the parity relations (5.12), (6.3) and (6.4), the spin mixing mechanism for the Kraus unnatural and natural transversity amplitudes reads

$$
U^J_{\lambda\tau}(\ell) = \sum_{K,\mu} < J\lambda|W_\ell|K\mu > U^K_{\mu\tau}(S) \tag{6.8}
$$

$$
N^J_{\lambda\tau}(\ell) = \sum_{K,\mu\neq 0} < J\lambda|W_\ell|K\mu > N^K_{\mu\tau}(S) \tag{6.9}
$$

where $U^K_{\lambda\tau}(S)$ and $N^K_{\lambda\tau}(S)$ are the S-matrix unnatural and natural exchange transversity amplitudes, respectively, and where

$$
< J\lambda|W_\ell|K\mu > = < p_c p_d; J\lambda, +|V_\ell|p_c p_d; K\mu, + > -i(-1)^\mu < p_c p_d; J\lambda, +|V_\ell|p_c p_d; K - \mu, - > \tag{6.10}
$$

We have omitted isospin labels in (6.10) for the sake of brevity. From the unitarity of the Kraus operators V_ℓ follows the unitarity of matrices $< J\lambda|W_\ell|K\mu >$ in (6.8)

$$
\sum_{K\mu} < J\lambda|W_\ell|K\mu >< J'\lambda'|W_\ell|K\mu >^* = \delta_{JJ'}\delta_{\lambda\lambda'} \tag{6.11}
$$

The expressions (6.8) and (6.9) satisfy the P-parity relations (6.7).

We now impose the conditions $\lambda - \chi = \mu - \xi$ that follow from the Lorentz symmetry of the Kraus operators. The expressions (6.8) and (6.9) then read

$$
\begin{aligned}
U^J_{\lambda\tau}(\ell) &= \sum_K < p_c p_d; J\lambda, +|V_\ell|p_c p_d; K\lambda, + > U^K_{\lambda\tau}(S) \\
&\quad -i < p_c p_d; J\lambda, +|V_\ell|p_c p_d; K\lambda - 1, - > U^K_{\lambda-1\tau}(S)
\end{aligned} \tag{6.12}
$$

$$N^J_{\lambda\tau}(\ell) = \sum_K < p_c p_d; J\lambda, +|V_\ell|p_c p_d; K\lambda, + > N^K_{\lambda\tau}(S) \quad (6.13)$$
$$+i < p_c p_d; J\lambda, +|V_\ell|p_c p_d; K\lambda - 1, - > N^K_{\lambda-1\tau}(S)$$

With a change $\lambda \to -\lambda$ the expressions (6.12) and (6.13) read

$$U^J_{-\lambda\tau}(\ell) = +(-1)^\lambda \Big(\sum_K < p_c p_d; J\lambda, +|V_\ell|p_c p_d; K\lambda, + > U^K_{\lambda\tau}(S) \quad (6.14)$$
$$-i < p_c p_d; J\lambda, +|V_\ell|p_c p_d; K\lambda + 1, - > U^K_{\lambda+1\tau}(S)\Big)$$

$$N^J_{-\lambda\tau}(\ell) = -(-1)^\lambda \Big(\sum_K < p_c p_d; J\lambda, +|V_\ell|p_c p_d; K\lambda, + > N^K_{\lambda\tau}(S) \quad (6.15)$$
$$+i < p_c p_d; J\lambda, +|V_\ell|p_c p_d; K\lambda + 1, - > N^K_{\lambda+1\tau}(S)\Big)$$

These expressions must satisfy the P-parity relations (6.7) for any λ such that $|\lambda| \leq J$. This is possible only when

$$< p_c p_d; J\lambda, +|V_\ell|p_c p_d; K\lambda + 1, - > = < p_c p_d; J\lambda, +|V_\ell|p_c p_d; K\lambda - 1, - > = 0 \quad (6.16)$$

Then the spin mechanism for the Kraus transversity amplitudes takes a simple form for any $|\lambda| \leq J$

$$U^J_{\lambda\tau}(\ell) = \sum_{K=J-1,J,J+1} < p_c p_d; J\lambda I_J, +|V_\ell|p_c p_d; K\lambda I_K, + > U^K_{\lambda\tau}(S) \quad (6.17)$$

$$N^J_{\lambda\tau}(\ell) = \sum_{K=J-1,J,J+1} < p_c p_d; J\lambda I_J, +|V_\ell|p_c p_d; K\lambda I_K, + > N^K_{\lambda\tau}(S) \quad (6.18)$$

where we have restored the isospin labels and used the constraint (5.6) from the Lorentz symmetry of Kraus operators. The Lorentz conditions (5.5) also require

$$< p_c p_d; J\lambda I_J, +|V_\ell|p_c p_d; K\lambda I_K, - > = 0 \quad (6.19)$$

With the only non-zero dephasing amplitudes given by the spin mixing mechanism (6.17) and (6.18), the spin mixing mechanism for the Kraus helicity amplitudes (4.32) reads

$$H^J_{\lambda\chi,0\nu}(\ell) = \sum_{K=J-1,J,J+1} < p_c p_d; J\lambda I_J, \chi|V_\ell|p_c p_d; K\lambda I_K, \chi > H^K_{\lambda\chi,0\nu}(S) \quad (6.20)$$

where

$$< p_c p_d; J\lambda I_J, +|V_\ell|p_c p_d; K\lambda I_K, + > = < p_c p_d; J\lambda I_J, -|V_\ell|p_c p_d; K\lambda I_K, - > \quad (6.21)$$

We have arrived at a self-consistent form of the spin mixing mechanism for Kraus amplitudes that arises from their consistency with the Standard Model. The consistency of the Kraus transversity amplitudes with Lorentz symmetry of the Kraus operators results in the breaking of the unitarity of the Kraus operators into a block unitarity with each block characterized by the helicity λ. The self-consistency constraint requires that the dephasing interaction with the environment conserves both the dipion helicity and the recoil nucleon helicity and transversity.

VII. THE MEASUREMENT OF THE OBSERVED FINAL STATE $\rho_f(O)$.

A. Angular intensities and density matrix elements

Assuming unitary S-matrix we used the evolution equation $\rho = S\rho_i S^+$ to develop in Ref [20] a spin formalism to express the measured final state $\rho_f(p_c p_d;\theta\phi,\vec{P})$ in terms of spin density matrix elements $(R^j_k)^{JJ'}_{\lambda\lambda'}$ which are bilinear combinations of helicity amplitudes $H^J_{\lambda\chi,0\nu}(S)$ or transversity amplitudes $U^J_{\lambda,\tau}(S)$ and $N^J_{\lambda,\tau}(S)$. On the other hand, the data analyses using this formalism lead to the conclusion that $\rho_f(p_c p_d;\theta\phi,\vec{P})$ is given by a non-unitary Kraus representation (4.24). Self-consistency requires that the Kraus representation preserves the experimental form of the angular intensities describing the final state density matrix and the form of the expressions for density matrix elements in terms of transversity amplitudes. In this Section we show that such invariance holds provided that the Kraus partial wave amplitudes satisfy the same parity relations as the S-matrix amplitudes due to the conservation of P-parity.

The form (4.25) of each term $\rho_f(\theta\phi,\vec{P};\ell\ell)$ in the Kraus representation (4,24) is the same as the form of the S-matrix final state in terms of S-matrix amplitudes

$$\rho_f(\theta\phi,\vec{P})_{\chi\chi'} = \sum_{\nu\nu'} S_{\chi,0\nu}(\theta\phi)\rho_b(\vec{P})_{\nu\nu'} S_{\chi',0\nu'}(\theta\phi) \tag{7.1}$$

We can thus apply the spin formalism develped in Ref. [20] to each such term separately. Then each term $\rho_f(\theta\phi,\vec{P};\ell\ell)$ in (4.29) has the following form in terms of angular intensities

$$\rho_f(\theta\phi,\vec{P};\ell\ell) = \frac{1}{2}\big(I^0(\theta\phi,\vec{P};\ell\ell)\sigma^0 + \vec{I}(\theta\phi,\vec{P};\ell\ell)\vec{\sigma}\big) \tag{7.2}$$

with a decomposition of the intensities $I^j(\theta\phi,\vec{P};\ell\ell)$ in terms of component intensities

$$I^j(\theta\phi,\vec{P};\ell\ell) = I^j_u(\theta\phi;\ell\ell) + P_x I^j_x(\theta\phi;\ell\ell) + P_y I^j_y(\theta\phi;\ell\ell) + P_z I^j_z(\theta\phi;\ell\ell) \tag{7.3}$$

The components of intensities $I^j_k(\theta\phi;\ell\ell)$ have angular expansion

$$I^j_k(\theta\phi;\ell\ell) = \sum_{J\lambda}\sum_{J'\lambda'} (R^j_k(\ell\ell))^{JJ'}_{\lambda\lambda'} Y^J_\lambda(\theta\phi) Y^{J'*}_{\lambda'}(\theta\phi) \tag{7.4}$$

where the matrix elements $(R^j_k(\ell\ell))^{JJ'}_{\lambda\lambda'}$ are expressed in terms of Kraus partial wave helicity amplitudes

$$(R^j_k(\ell\ell))^{JJ'}_{\lambda\lambda'} = \frac{1}{2}\sum_{\chi,\chi'}\sum_{\nu\nu'} (\sigma^j)_{\chi'\chi} H^J_{\lambda\chi,0\nu}(\ell)(\sigma_k)_{\nu\nu'} H^{J'*}_{\lambda'\chi',0\nu'}(\ell) \tag{7.5}$$

As the result of linearity of the Kraus representation, the observed final state density matrix has the same form as in the case of the S-matrix

$$\rho_f(\theta\phi,\vec{P}) = \frac{1}{2}\big(I^0(\theta\phi,\vec{P})\sigma^0 + \vec{I}(\theta\phi,\vec{P})\vec{\sigma}\big) \tag{7.6}$$

where

$$I^j_k(\theta\phi) = \sum_{\ell=1}^{4} p_{\ell\ell} I^j_k(\theta\phi;\ell\ell) \tag{7.7}$$

To bring $I_k^j(\theta\phi)$ to the experimental S-matrix form we need to bring to this form the intensities $I_k^j(\theta\phi;\ell\ell)$. To this end the density matrix elements $(R_k^j(\ell\ell))^{JJ'}_{\lambda\lambda'}$ must satisfy the same parity relations as the corresponding matrix elements in the S-matrix theory. This in turn requires that the Kraus partial wave amplitudes satisfy the same parity relations as the corresponding S-matrix partial wave amplitudes. With the parity relation (5.12) the components $I_k^j(\theta\phi;\ell\ell)$ then have the desired form [20]

$$I_k^j(\theta\phi;\ell\ell) = \sum_{J\lambda}\sum_{J'\lambda'}(ReR_k^j(\ell\ell))^{JJ'}_{\lambda\lambda'}Re(Y^J_\lambda(\theta\phi)Y^{J'*}_{\lambda'}(\theta\phi)) \tag{7.8}$$

for $(k,j) = (u,0),(y,0),(u,2),(y,2),(x,1),(z,1),(x,3),(z,3)$ and the form

$$I_k^j(\theta\phi;\ell\ell) = -\sum_{J\lambda}\sum_{J'\lambda'}(ImR_k^j(\ell\ell))^{JJ'}_{\lambda\lambda'}Im(Y^J_\lambda(\theta\phi)Y^{J'*}_{\lambda'}(\theta\phi)) \tag{7.9}$$

for $(x,0),(z,0),(x,2),(z,2),(u,1),(y,1),(u,3),(y,3)$. From (7.7) it follows that the measured intensities $I_k^j(\theta\phi)$ will retain the same form of angular expansions provided that the measured density matrix elements $(R_k^j)^{JJ}_{\lambda\lambda'}$ are redefined as

$$(R_k^j)^{JJ}_{\lambda\lambda'} = \sum_{\ell=1}^{4} p_{\ell\ell}(R_k^j(\ell\ell))^{JJ'}_{\lambda\lambda'} \tag{7.10}$$

The measured density matrix elements are thus environment-averaged elements of the Kraus density matrices.

B. Effective amplitudes in $\pi N \to \pi\pi N$

The Kraus density matrix elements $(R_k^j(\ell\ell))^{JJ'}_{\lambda\lambda'}$ are linear combinations of bilinear terms of Kraus transversity amplitudes which are the same as those for the S-matrix elements and which are given in the Table I. The measured density matrix elements $(R_k^j)^{JJ}_{\lambda\lambda'}$ are given by (5.10) in terms of elements of Kraus density matrices. Because this relation is linear the measured elements $(R_k^j)^{JJ}_{\lambda\lambda'}$ will retain the form given in Table I. with moduli and bilinear terms of transversity amplitudes now replaced by environment-averaged moduli and bilinear terms of Kraus amplitudes, respectively. The expressions for the measured bilinear terms for any two transversity amplitudes A and B read

$$\begin{aligned} <|A|^2> &= \sum_{\ell=1}^{4} p_{\ell\ell}|A(\ell)|^2 \\ Re<AB^*> &= \sum_{\ell=1}^{4} p_{\ell\ell}Re(A(\ell)B^*(\ell)) \\ Im<AB^*> &= \sum_{\ell=1}^{4} p_{\ell\ell}Im(A(\ell)B^*(\ell)) \end{aligned} \tag{7.11}$$

where we have omitted the spin labels of the amplitudes for the sake of clarity. Note that the symbol AB^* does not stand for a product of two complex functions A and B^* but communicates which amplitudes $A(\ell)B^*(\ell)$ are being averaged. The averaged bilinear terms can be written in terms of effective transversity amplitudes

$$\begin{aligned} <|A|^2> &= |A|^2 \\ Re<AB^*> &= |A||B|\cos\Phi(AB^*) \\ Im<AB^*> &= |A||B|\sin\Psi(AB^*) \end{aligned} \tag{7.12}$$

TABLE I: Density matrix elements expressed in terms of nucleon transversity amplitudes with definite t-channel naturality. The spin indices JJ' which always go with helicities $\lambda\lambda'$ have been omitted in the amplitudes. The coefficients $\eta_\lambda = 1$ for $\lambda = 0$ and $\eta_\lambda = 1/\sqrt{2}$ for $\lambda \neq 0$. Table from Ref. [20].

$(R_u^0)^{JJ'}_{\lambda\lambda'}$	$\eta_\lambda\eta_{\lambda'}[U_{\lambda,u}U^*_{\lambda',u} + N_{\lambda,u}N^*_{\lambda',u} + U_{\lambda,d}U^*_{\lambda',d} + N_{\lambda,d}N^*_{\lambda',d}]$
$(R_y^0)^{JJ'}_{\lambda\lambda'}$	$\eta_\lambda\eta_{\lambda'}[U_{\lambda,u}U^*_{\lambda',u} + N_{\lambda,u}N^*_{\lambda',u} - U_{\lambda,d}U^*_{\lambda',d} - N_{\lambda,d}N^*_{\lambda',d}]$
$(R_x^0)^{JJ'}_{\lambda\lambda'}$	$-i\eta_\lambda\eta_{\lambda'}[U_{\lambda,u}N^*_{\lambda',d} + N_{\lambda,u}U^*_{\lambda',d} - U_{\lambda,d}N^*_{\lambda',u} - N_{\lambda,d}U^*_{\lambda',u}]$
$(R_z^0)^{JJ'}_{\lambda\lambda'}$	$\eta_\lambda\eta_{\lambda'}[U_{\lambda,u}N^*_{\lambda',d} + N_{\lambda,u}U^*_{\lambda',d} + U_{\lambda,d}N^*_{\lambda',u} + N_{\lambda,d}U^*_{\lambda',u}]$
$(R_u^2)^{JJ'}_{\lambda\lambda'}$	$-\eta_\lambda\eta_{\lambda'}[U_{\lambda,u}U^*_{\lambda',u} - N_{\lambda,u}N^*_{\lambda',u} - U_{\lambda,d}U^*_{\lambda',d} + N_{\lambda,d}N^*_{\lambda',d}]$
$(R_y^2)^{JJ'}_{\lambda\lambda'}$	$-\eta_\lambda\eta_{\lambda'}[U_{\lambda,u}U^*_{\lambda',u} - N_{\lambda,u}N^*_{\lambda',u} + U_{\lambda,d}U^*_{\lambda',d} - N_{\lambda,d}N^*_{\lambda',d}]$
$(R_x^2)^{JJ'}_{\lambda\lambda'}$	$i\eta_\lambda\eta_{\lambda'}[U_{\lambda,u}N^*_{\lambda',d} - N_{\lambda,u}U^*_{\lambda',d} + U_{\lambda,d}N^*_{\lambda',u} - N_{\lambda,d}U^*_{\lambda',u}]$
$(R_z^2)^{JJ'}_{\lambda\lambda'}$	$-\eta_\lambda\eta_{\lambda'}[U_{\lambda,u}N^*_{\lambda',d} - N_{\lambda,u}U^*_{\lambda',d} - U_{\lambda,d}N^*_{\lambda',u} + N_{\lambda,d}U^*_{\lambda',u}]$
$(R_u^1)^{JJ'}_{\lambda\lambda'}$	$-i\eta_\lambda\eta_{\lambda'}[U_{\lambda,u}N^*_{\lambda',u} - N_{\lambda,u}U^*_{\lambda',u} - U_{\lambda,d}N^*_{\lambda',d} + N_{\lambda,d}U^*_{\lambda',d}]$
$(R_y^1)^{JJ'}_{\lambda\lambda'}$	$-i\eta_\lambda\eta_{\lambda'}[U_{\lambda,u}N^*_{\lambda',u} - N_{\lambda,u}U^*_{\lambda',u} + U_{\lambda,d}N^*_{\lambda',d} - N_{\lambda,d}U^*_{\lambda',d}]$
$(R_x^1)^{JJ'}_{\lambda\lambda'}$	$-\eta_\lambda\eta_{\lambda'}[U_{\lambda,u}U^*_{\lambda',d} - N_{\lambda,u}N^*_{\lambda',d} + U_{\lambda,d}U^*_{\lambda',u} - N_{\lambda,d}N^*_{\lambda',u}]$
$(R_z^1)^{JJ'}_{\lambda\lambda'}$	$-i\eta_\lambda\eta_{\lambda'}[U_{\lambda,u}U^*_{\lambda',d} - N_{\lambda,u}N^*_{\lambda',d} - U_{\lambda,d}U^*_{\lambda',u} + N_{\lambda,d}N^*_{\lambda',u}]$
$(R_u^3)^{JJ'}_{\lambda\lambda'}$	$\eta_\lambda\eta_{\lambda'}[U_{\lambda,u}N^*_{\lambda',u} + N_{\lambda,u}U^*_{\lambda',u} + U_{\lambda,d}N^*_{\lambda',d} + N_{\lambda,d}U^*_{\lambda',d}]$
$(R_y^3)^{JJ'}_{\lambda\lambda'}$	$\eta_\lambda\eta_{\lambda'}[U_{\lambda,u}N^*_{\lambda',u} + N_{\lambda,u}U^*_{\lambda',u} - U_{\lambda,d}N^*_{\lambda',d} - N_{\lambda,d}U^*_{\lambda',d}]$
$(R_x^3)^{JJ'}_{\lambda\lambda'}$	$-i\eta_\lambda\eta_{\lambda'}[U_{\lambda,u}U^*_{\lambda',d} + N_{\lambda,u}N^*_{\lambda',d} - U_{\lambda,d}U^*_{\lambda',u} - N_{\lambda,d}N^*_{\lambda',u}]$
$(R_z^3)^{JJ'}_{\lambda\lambda'}$	$\eta_\lambda\eta_{\lambda'}[U_{\lambda,u}U^*_{\lambda',d} + N_{\lambda,u}N^*_{\lambda',d} + U_{\lambda,d}U^*_{\lambda',u} + N_{\lambda,d}N^*_{\lambda',u}]$

where $|A|$ and $|B|$ are the moduli of the effective amplitudes and $\cos\Phi(AB^*)$ and $\sin\Psi(AB^*)$ are their correlations. With this form of measured bilinear terms we can no longer associate complex functions A and B with the moduli $|A|$ and $|B|$, respectively, and identify the phases $\Phi(AB^*)$ and $\Psi(AB^*)$ as relative phases of these complex functions. In general the measured correlations violate the necessary trigonometric identity with $\cos^2\Phi(AB^*) + \sin^2\Psi(AB^*) \neq 1$. Since the phases $\Phi(AB^*)$ are not relative phases of complex functions, they violate the phase condition

$$\Phi(AB^*) = \Phi(AC^*) + \Phi(CB^*) \tag{7.13}$$

and the equivalent cosine condition

$$\cos^2\Phi(AB^*) + \cos^2\Phi(AC^*) + \cos^2\Phi(CB^*) - 2\cos\Phi(AB^*)\cos\Phi(AC^*)\cos\Phi(CB^*) = 1 \tag{7.14}$$

which hold for any three complex valued functions A, B, C.

To better understand the meaning of the measured moduli and correlations we define Kraus vectors in the Hilbert space $H(E)$

$$|A> = \sum_{\ell=1}^{4} \sqrt{p_{\ell\ell}} A(\ell)|e_\ell> \tag{7.15}$$

and a scalar product

$$<A|B> = \sum_{\ell=1}^{4} p_{\ell\ell} A(\ell) B^*(\ell) \tag{7.16}$$

Comparing (5.16) with (5.11) and (5.12) we find

$$\begin{aligned} |A|^2 &= <A|A> \\ |A||B|\cos\Phi(AB^*) &= Re <A|B> \\ |A||B|\sin\Psi(AB^*) &= Im <A|B> \end{aligned} \tag{7.17}$$

These relations provide a new physical interpretation of the measured (averaged) bilinear terms: instead of associating the measured effective amplitudes with complex functions we must associate them with complex Kraus vectors (5.15). The measured bilinear terms of effective amplitudes are real or imaginary parts of scalar products of their Kraus vectors.

It should be noted that the bilinear terms of Kraus amplitudes $A(\ell)B^*(\ell)$ carry the quantum numbers of the state vectors $|e_\ell>$ which define the interacting degrees of freedom of the quantum environment. The bilinear terms of effective amplitudes are mixtures of these quantum numbers.

C. Conservation of probability

The unitarity of the dephasing amplitudes and the spin mixing mechanism (6.17),(6.18) imply that for each $\ell = 1, 4$ and $\tau = u, d$

$$\sum_{J=\lambda}^{J_{max}} |U^J_{\lambda\tau}(\ell)|^2 = \sum_{K=\lambda}^{J_{max}} |U^K_{\lambda\tau}(S)|^2 \tag{7.18}$$

$$\sum_{J=\lambda}^{J_{max}} |N^J_{\lambda\tau}(\ell)|^2 = \sum_{K=\lambda}^{J_{max}} |N^K_{\lambda\tau}(S)|^2 \tag{7.19}$$

where $J_{max} = J_{max}(m)$ depends on the dipion mass m. The observed moduli for transversity amplitudes are given by

$$\begin{aligned} |U^J_{\lambda\tau}|^2 &= \sum_{\ell=1}^{4} p_{\ell\ell} |U^J_{\lambda\tau}(\ell)|^2 \\ |N^J_{\lambda\tau}|^2 &= \sum_{\ell=1}^{4} p_{\ell\ell} |N^J_{\lambda\tau}(\ell)|^2 \end{aligned} \tag{7.20}$$

It follows from (7.18) and (7.19) that the observed differential cross-section is equal to that given by the S-matrix amplitudes

$$\frac{d^2\sigma}{dmdt} = \sum_{J\lambda,\tau} |U^J_{\lambda\tau}|^2 + |N^J_{\lambda\tau}|^2 = \sum_{J\lambda,\tau} |U^J_{\lambda\tau}(S)|^2 + |N^J_{\lambda\tau}(S)|^2 \tag{7.21}$$

This relation embodies the conservation of probability of a particle reaction in dephasing interaction with the environment. The spin mixing mechanism (6.20) for the helicity amplitudes implies the same conservation of the differential cross-section.

We note that the the observed polarized target assymetry is similarly given by the S-matrix amplitudes

$$\begin{aligned} T\frac{d^2\sigma}{dmdt} &= \sum_{J\lambda} |U^J_{\lambda u}|^2 + |N^J_{\lambda u}|^2 - |U^J_{\lambda d}|^2 - |N^J_{\lambda d}|^2 \\ &= \sum_{J\lambda} |U^J_{\lambda u}(S)|^2 + |N^J_{\lambda u}(S)|^2 - |U^J_{\lambda d}(S)|^2 - |N^J_{\lambda d}(S)|^2 \end{aligned} \tag{7.22}$$

and is conserved by the dephasing interaction. More generally, for two unnatural exchange or two natural exchange transversity amplitudes we have

$$\sum_{J=\lambda}^{J_{max}} A^J_{\lambda\tau} B^{J*}_{\lambda\tau} = \sum_{K=\lambda}^{J_{max}} A^K_{\lambda\tau}(S) B^{K*}_{\lambda\tau}(S) \tag{7.23}$$

Recall that unitary evolution law of the S-matrix imposes relative phases $(n(A) - n(B))\pi$ on such transversity amplitudes where $n(A), n(B)$ are integers [20]. Then the equations (7.23) imply a sum rule

$$\sum_{J=\lambda}^{J_{max}} |A^J_{\lambda\tau}||B^J_{\lambda\tau}| \sin\Psi(A^J_{\lambda\tau}B^J_{\lambda\tau}) = 0 \tag{7.24}$$

D. The measurement of the Kraus amplitudes

Kraus amplitudes are complex valued functions that do not seem to be accessible to a direct measurement. A possible approach employed by past amplitude analyses is to impose analyticity on the measured data. In the case of the S-and P-wave subsystem in $\pi^- p \to \pi^-\pi^+ n$ the two sets of equations for transversity amplitudes with transversities "up" and "down", respectively, are supplemented by the cosine conditions (7.14) [19, 21]. There are four solutions of the two sets for the transversity amplitudes that can be identified with the four sets of the S- and P-wave Kraus transversity amplitudes. Using a model for the experimental determination of the probabilities $p_{\ell\ell}$ we show in Ref. [21] that the resulting Kraus representation provides an excellent fit to all observed data - without any free parameters. On this basis we set the dimension $M = \dim H(E) = 4$.

VIII. THE PREDICTION OF THE $\rho^0(770) - f_0(980)$ MIXING IN $\pi^- p \to \pi^-\pi^+ n$.

A. Resonance mixing in $\pi^- p \to \pi^-\pi^+ n$ below 1400 MeV

We consider CERN measurements of $\pi^- p \to \pi^-\pi^+ n$ on polarized target in the dipion mass range 580-1400 MeV at low momentum transfers $0.005 < |t| < 0.20$ (Gev/c)^2 [8, 9] and high momentum transfers $0.20 < |t| < 1.0$ (GeV/c)^2 [10]. It is useful to divide the mass range into two intervals 580-1080 MeV and 960-1400 MeV.

In the first mass interval at low t the S- and P-waves dominate and D-waves are neglected. The amplitudes $S_\tau(\ell)$ and $L_\tau(\ell)$ are the S-wave and P-wave unnatural Kraus transversity amplitudes with helicity $\lambda = 0$. The amplitudes $U_\tau(\ell)$ and $N_\tau(\ell)$ are the P-wave unnatural and natural exchange Kraus transversity amplitudes with helicity $\lambda = 1$. The amplitudes $S_\tau(\ell)$ and $L_\tau(\ell)$ form a dephasing doublet

$$\begin{aligned} L_\tau(\ell) &= V^0_{10}(\ell)S_\tau(S) + V^0_{11}(\ell)L_\tau(S) \\ S_\tau(\ell) &= V^0_{00}(\ell)S_\tau(S) + V^0_{01}(\ell)L_\tau(S) \end{aligned} \tag{8.1}$$

where $V^\lambda_{JK}(\ell) =< J\lambda + |V_\ell|K\lambda+ >$ is a unitary $U(2)$ matrix and $S_\tau(S), L_\tau(S)$ are S-matrix amplitudes. The amplitudes $U_\tau(\ell)$ and $N_\tau(\ell)$ are dephasing singlets

$$\begin{gathered} U_\tau(\ell) = V^1_{11}(\ell)U_\tau(S) \\ N_\tau(\ell) = V^1_{11}(\ell)N_\tau(S) \end{gathered} \tag{8.2}$$

where $V^1_{11}(\ell)$ is a unitary $U(1)$ matrix. Thus we predict $\rho^0(770) - f_0(980)$ mixing in the amplitudes $S_\tau(\ell)$ and $L_\tau(\ell)$.

In the following we omit the label ℓ for the sake of brevity. In the first mass interval at high t the D-waves cannot be neglected. There are three unnatural Kraus transversity amplitudes $D^0_\tau, D^U_\tau, D^{2U}_\tau$ corresponding to the helicities $\lambda = 0, 1, 2$ and two natural Kraus transversity amplitudes D^N_τ, D^{2N}_τ corresponding to helicities $\lambda = 1, 2$. The three helicity zero amplitudes S_τ, L_τ, D^0_τ

form a dephasing triplet

$$\begin{aligned} D_\tau^0 &= V_{20}^0 S_\tau(S) + V_{21}^0 L_\tau(S) + V_{22}^0 D_\tau^0(S) \\ L_\tau &= V_{10}^0 S_\tau(S) + V_{11}^0 L_\tau(S) + V_{12}^0 D_\tau^0(S) \\ S_\tau &= V_{00}^0 S_\tau(S) + V_{01}^0 L_\tau(S) + V_{02}^0 D_\tau^0(S) \end{aligned} \tag{8.3}$$

where V_{JK}^0 is a unitary $U(3)$ matrix. Its most general form is the CKM matrix [34, 35]

$$V_{JK}^0 = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{8.4}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. Lorentz symmetry of Kraus operators imposes a constraint $K = J-1, J, J+1$. This means that $V_{20}^0 = V_{02}^0 = 0$. From the CKM matrix it follows that $c_{13} = 0$ so that $V_{21}^0 = V_{12}^0 = 0$. With $s_{13} = \pm 1$ the dephasing amplitude $V_{22}^0 = \pm e^{-i\delta}$ is effectively a dephasing singlet and D_τ^0 decouples from the S_τ and L_τ doublet. Thus we expect no $\rho^0(770)$ and $f_0(980)$ mixing in the D_τ^0 amplitudes which is in fact observed. However note that the mixing of the S and L amplitudes still depends on the phase δ.

The amplitudes U_τ, D_τ^U and N_τ, D_τ^N form two dephasing doublets. Since there is no experimental evidence for $\rho^0(770)$ in D_τ^U the matrix V_{JK}^1 must be a diagonal unitary matrix. This predicts no $\rho^0(770)$ in the amplitudes D_τ^N which is observed. The amplitudes U_τ^{2U} and D_τ^{2N} form two dephasing singlets and show no $\rho^0(770)$ mixing, as predicted by the spin mixing mechanism..

In the second mass interval D-waves contribute also at low t although D_τ^{2U} and D^{2N} can still be neglected at low t. There are two resonances in this mass interval: $f_2(1270)$ which dominates in D_τ^0 and $f_0(1370)$ which dominates in S_τ [36]. We do not expect $f_2(1270)$ to mix with L_τ because D_τ^0 still decouples from the S- and L-wave amplitudes. The matrix V_{JK}^1 is still diagonal in this mass interval as D_τ^N shows the presence of $f_2(1270)$ while this resonance does not mix with N_τ. This V_{JK}^1 is in agreement with the behavior of U_τ and D_τ^U.

No significant contribution of $f_0(1370)$ is observed in the amplitude L_τ which is suppressed. This suggests that in the mass region of the $f_0(1370)$ resonance the element $V_{10}^0 \approx 0$. The CKM matrix then implies

$$\begin{array}{ll} ReV_{10}^0 = -s_{12}c_{23} - c_{12}s_{23}s_{13}\cos\delta \approx 0, & ImV_{10}^0 = -c_{12}s_{23}s_{13}\sin\delta \approx 0 \\ ReV_{01}^0 \approx -c_{12}s_{23}(1-\cos^2\delta), & ImV_{01}^0 = -s_{12}c_{23}s_{13}\sin\delta \end{array} \tag{8.5}$$

This suggests that $\delta \approx 0$ so that both off-diagonal elements are small in this mass region.

We conclude that below 1400 MeV there is no resonance mixing in $\pi^- p \to \pi^-\pi^+ n$ other than the $\rho^0(770) - f_0(980)$ mixing. We now turn to the derivation of the predictions for the moduli and relative phases of the amplitudes S_τ and L_τ from the spin mixing mechanism.

B. Phases of the S-matrix amplitudes

To study the observable effects of the spin mixing mechanism we need to introduce the phases of the S-matrix transversity amplitudes. In this subsection we omit the label S of the S-matrix transversity amplitudes. In the Part I. of this work [20] we have shown that the unitary evolution law imposes a constraint on relative phases

$$\Phi(U_{\lambda\tau}^J) - \Phi(N_{\lambda'-\tau}^{J'}) = 0, \pm\pi, \pm 2\pi \tag{8.6}$$

for all J, J' and corresponding λ, λ'. It follows from the unitary conditions (8.6) that all unnatural exchange amplitudes $U^J_{\lambda\tau}$ as well as all natural exchange amplitudes $N^J_{\lambda\tau}$ share the same absolute phase up to an integer multiple of π

$$\begin{aligned} \Phi(U^J_{\lambda\tau}) &= \Phi(S_\tau) + \pi n(U^J_{\lambda\tau}) \\ \Phi(N^J_{\lambda\tau}) &= \Phi(N_\tau) + \pi n(N^J_{\lambda\tau}) \end{aligned} \tag{8.7}$$

where $n(U^J_{\lambda\tau})$ and $n(N^J_{\lambda\tau})$ are integers. The phases $\Phi(S_\tau)$ and $\Phi(N_\tau)$ depend on s, t, m.

The unitary relative phases imply that if there is a resonance at mass m_R in a partial wave with $J = J_R$, then all contributing partial waves will have the same resonant phase near m_R. Mathematixcally, this does not necessarily imply that any of the partial waves with $J \neq J_R$ must resonate and show a resonant peak or a dip since the non-resonant moduli can still have resonant phases. Although the conditions (8.6) appear to allow such resonance mixing, it is excluded by the the symmetries of the S-matrix. Indeed, the production as well as the decay of a resonance is described entirely by the resonant S-matrix amplitude $A^J_{\lambda\tau}(S)$ while the mixing of resonances in the Kraus amplitudes $A^J_{\lambda\tau}(\ell)$ arises entirely from the dephasing amplitudes.

With (8.7) the S-matrix transversity amplitudes then have a form

$$\begin{aligned} U^J_{\lambda\tau} &= e^{i\Phi(S_\tau)} e^{i\pi n(U^J_{\lambda\tau})} |U^J_{\lambda\tau}| \\ N^J_{\lambda\tau} &= e^{i\Phi(N_\tau)} e^{i\pi n(N^J_{\lambda\tau})} |N^J_{\lambda\tau}| \end{aligned} \tag{8.8}$$

A self-consistent set of relative phases is presented in the Table II. in the Part I. of this work [20]. For the S- and P-wave subsystem below 980 MeV the relative phases read

$$\Phi(L^0_\tau) - \Phi(S^0_\tau) \;= 0 \tag{8.9a}$$
$$\Phi(L^0_\tau) - \Phi(U^0_\tau) \;= +\pi \tag{8.9b}$$
$$\Phi(U^0_\tau) - \Phi(S^0_\tau) \;= -\pi \tag{8.9c}$$

For masses $m \gtrsim 980$ MeV there is a change of sign in the interference terms of the amplitudes prompting a change of relative phases for $\tau = d$ [20]

$$\Phi(L^0_d) - \Phi(S^0_d) \;= +\pi \tag{8.10a}$$
$$\Phi(L^0_d) - \Phi(U^0_d) \;= +\pi \tag{8.10b}$$
$$\Phi(U^0_d) - \Phi(S^0_d) \;= 0 \tag{8.10c}$$

In (8.9) and (8.10) we have introduced the superscript 0 to label S- and P-wave S-matrix amplitudes to be used in the next subsection. It follows that for $m < 980$ MeV

$$S^0_\tau \;=\; +e^{i\Phi(S_\tau)}|S^0_\tau| \tag{8.11a}$$
$$L^0_\tau \;=\; +e^{i\Phi(S_\tau)}|L^0_\tau| \tag{8.11b}$$
$$U^0_\tau \;=\; -e^{i\Phi(S_\tau)}|U^0_\tau| \tag{8.11c}$$

while for the masses above 980 MeV the signs of L^0_d and U^0_d are reversed.

C. Predictions arising from the $\rho^0(770) - f_0(980)$ mixing

The mixing of the S-matrix amplitudes S^0_τ and L^0_τ given by the unitary transform (8.1) leads to observable modifications of the moduli and relative phases of these amplitudes. We shall assume that in the $\rho^0(770)$ mass region the P-wave amplitude $|L^0_\tau|$ resonates and dominates the flat and

small S-wave amplitude $|S_\tau^0|$. In the $f_0(980)$ mass region we assume that $|L_\tau^0|$ is small ≈ 0 and decreasing with mass while $|S_d^0|$ is larger and increasing as it emerges from a dip at 980 MeV. The phases of the amplitudes are the unitary phases (8.9) and (8.10).

The spin mixing matrix V_{JK}^0 is a unitary $U(2)$ matrix. The most general $U(2)$ matrix has the form [34]

$$V_{JK}^0 = \begin{pmatrix} +e^{i\phi_1}\cos\theta & e^{i(\phi_1+\phi_2)}\sin\theta \\ -e^{i\phi_2}\sin\theta & e^{i(\phi_2+\phi_3)}\cos\theta \end{pmatrix} \tag{8.12}$$

With $V_{11}^1 = e^{i\delta(U)}$ the spin mixing mechanism reads

$$\begin{aligned} L_\tau &= e^{i\phi_1}\big(+\cos\theta S_\tau^0 + e^{i\phi_2}\sin\theta L_\tau^0\big) && (8.13a)\\ S_\tau &= e^{i\phi_2}\big(-\sin\theta S_\tau^0 + e^{i\phi_3}\cos\theta L_\tau^0\big) && (8.13b)\\ U_\tau &= e^{i\delta(U)}U_\tau^0 && (8.13c) \end{aligned}$$

First we consider the $\rho^0(770)$ mass region and $m < 980$ MeV. With the phases (8.11) the equations (8.13) read

$$\begin{aligned} L_\tau &= +e^{i\Phi(S_\tau)}e^{i\phi_1}\big(+\cos\theta|S_\tau^0| + e^{i\phi_2}\sin\theta|L_\tau^0|\big) && (8.14a)\\ S_\tau &= +e^{i\Phi(S_\tau)}e^{i\phi_2}\big(-\sin\theta|S_\tau^0| + e^{i\phi_3}\cos\theta|L_\tau^0|\big) && (8.14b)\\ U_\tau &= -e^{i\Phi(S_\tau)}e^{i\delta(U)}|U_\tau^0| && (8.14c) \end{aligned}$$

The moduli have a form

$$\begin{aligned} |L_\tau|^2 &= \sin^2\theta|L_\tau^0|^2\big(1 + 2\cot\theta\cos\phi_2 X_\tau + \cot^2\theta X_\tau^2\big) && (8.15)\\ |S_\tau|^2 &= \cos^2\theta|L_\tau^0|^2\big(1 - 2\tan\theta\cos\phi_3 X_\tau + \tan^2\theta X_\tau^2\big) \\ |U_\tau|^2 &= |U_\tau^0|^2 \end{aligned}$$

where $X_\tau = |S_\tau^0|/|L_\tau^0|$. To fix the phases $\phi_i, i = 1,3$ we require that the observed relative phase $\cos\Phi_\tau(LU)$ is near the unitary value from (8.9) $\cos\Phi_\tau^0(LS) = +1$

$$\cos\Phi_\tau(LS) = \frac{Re(L_\tau S_\tau^*)}{|L_\tau||S_\tau|} = 1 - \epsilon \tag{8.16}$$

From (8.14) and (8.15) we find

$$\begin{aligned} \cos\Phi_\tau(LS) &= \frac{\cos(\phi_1-\phi_3)\sin\theta\cos\theta|L_\tau^0|^2(1+...)}{|\sin\theta||\cos\theta||L_\tau^0|^2(1+...)} && (8.17)\\ &= \cos(\phi_1-\phi_3)(1-\epsilon) \end{aligned}$$

where we have assumed $\sin\theta > 0, \cos\theta > 0$. The consistency with the unitary value requires that $\phi_1 = \phi_3 = \phi$. The unitarity of the resulting V_{JK}^0 requires $\phi_2 = \phi$. Similarly we require that

$$\cos\Phi_\tau(LU) = \frac{Re(L_\tau U_\tau^*)}{|L_\tau||U_\tau|} = -1 + \epsilon \tag{8.18}$$

is near the unitary value $\cos\Phi_\tau^0(LU) = -1$. This consistency implies that $\delta(U) = 2\phi$. The final form of the spin mixing mechanism below 980 MeV then reads

$$\begin{aligned} L_\tau &= +e^{i\Phi(S_\tau)}e^{i\phi}\big(+\cos\theta|S_\tau^0| + e^{i\phi}\sin\theta|L_\tau^0|\big) && (8.19)\\ S_\tau &= +e^{i\Phi(S_\tau)}e^{i\phi}\big(-\sin\theta|S_\tau^0| + e^{i\phi}\cos\theta|L_\tau^0|\big) && (8.20)\\ U_\tau &= -e^{i\Phi(S_\tau)}e^{i2\phi}|U_\tau^0| && (8.21) \end{aligned}$$

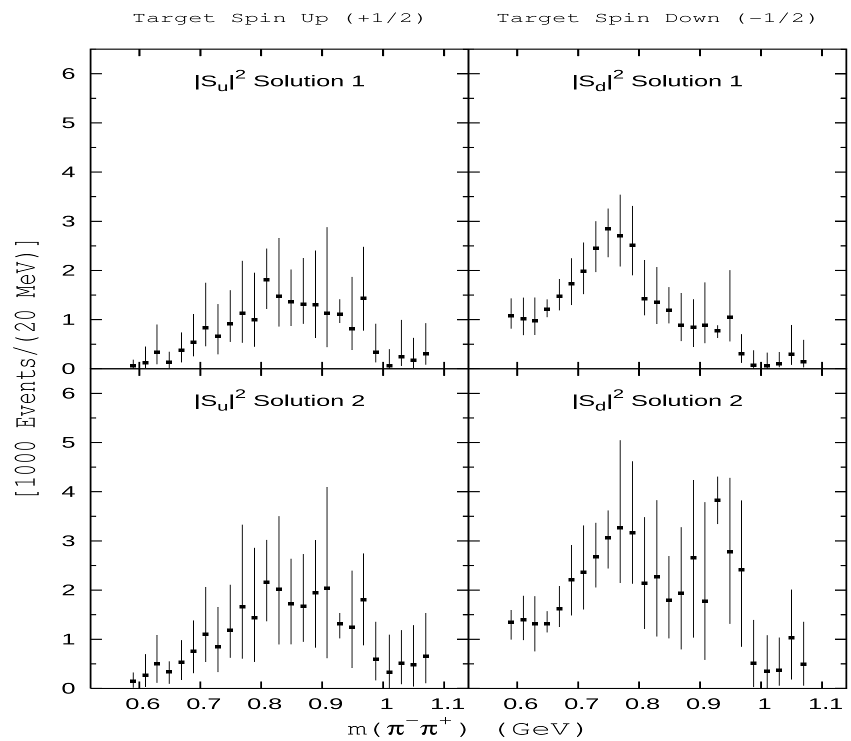

FIG. 1: The moduli of S-wave transversity amplitudes $|S_u|^2$ and $|S_d|^2$ from the analysis [17, 19].

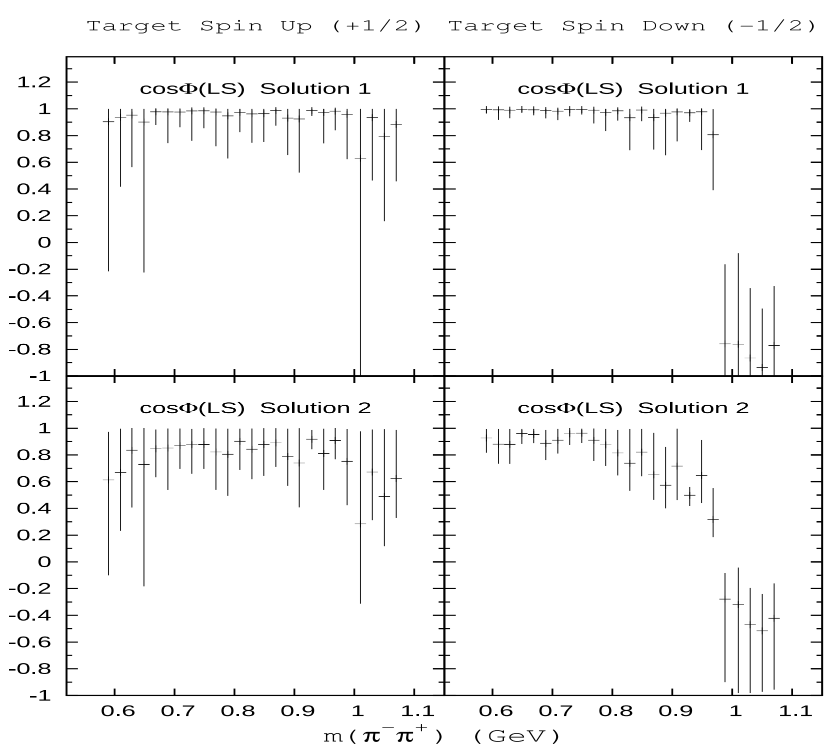

FIG. 2: Cosines $\cos\Phi_\tau(LS)$ from the amplitude analysis [17, 19].

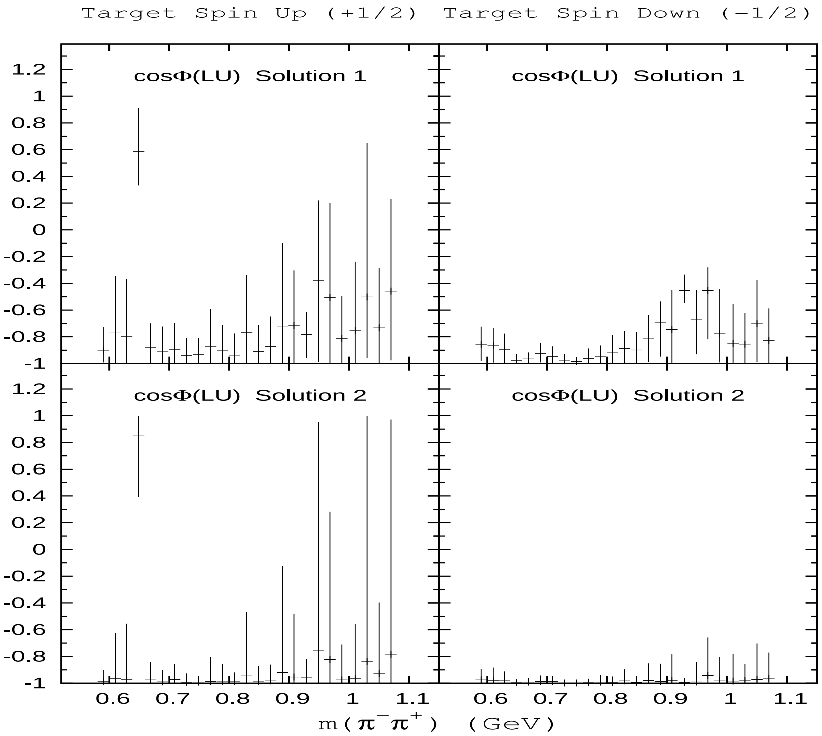

FIG. 3: Cosines $\cos\Phi_\tau(LU)$ from the amplitude analysis [17, 19].

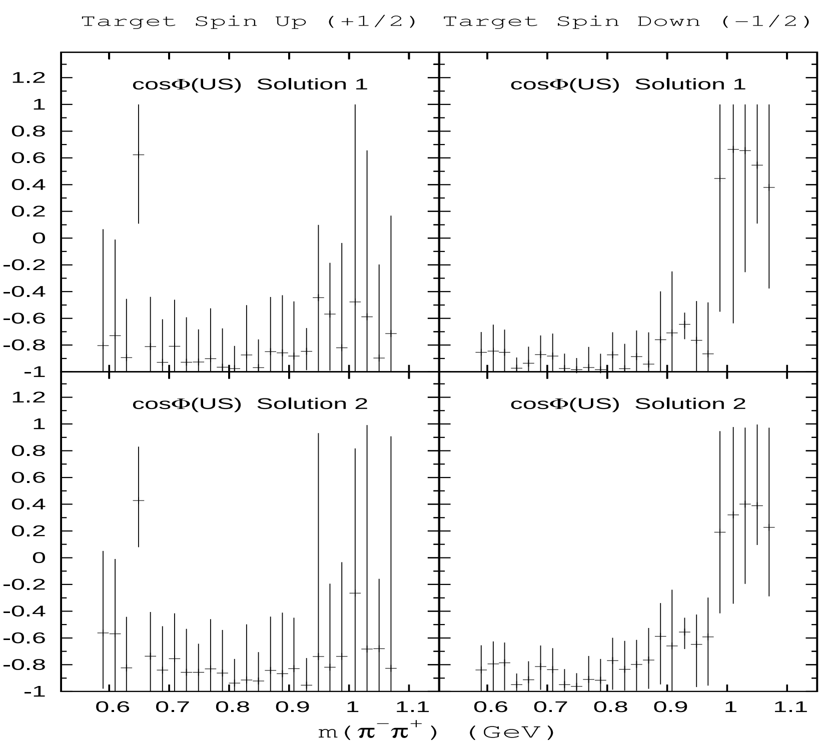

FIG. 4: Cosines $\cos\Phi_\tau(US)$ from the amplitude analysis [17, 19].

With $\phi_2 = \phi_3 = \phi$ in (8.15) the moduli predict the presence of $\rho^0(770)$ resonance in the amplitudes $|S_\tau|^2$. This prediction is confirmed by the amplitude analysis of the CERN data [17, 19] with the results for $|S_\tau|^2$ shown in the Figure 1. Near $\rho^0(770)$ mass $X_\tau \ll 1$ so that

$$\begin{aligned} |L_\tau|^2 &\approx \sin^2\theta|L_\tau^0|^2 \\ |S_\tau|^2 &\approx \cos^2\theta|L_\tau^0|^2 \end{aligned} \tag{8.22}$$

At the $m = 0.769$ MeV the ratio $\tan^2\theta = |L_d|^2/|S_d|^2 = 4.940$ which gives $\theta = 65.78°$ at this mass. Calculations of the cosines and sines of the relative phases up to the leading order in X_τ predict

$$\cos\Phi_\tau(LS) = \frac{Re(L_\tau S_\tau^*)}{|L_\tau||S_\tau|} = +1 - \frac{1}{2}\Big(\frac{2\sin\phi}{\sin 2\theta}X_\tau\Big)^2 \tag{8.23}$$

$$\sin\Phi_\tau(LS) = \frac{Im(L_\tau S_\tau^*)}{|L_\tau||S_\tau|} = -\frac{2\sin\phi}{\sin 2\theta}X_\tau$$

$$\cos\Phi_\tau(LU) = \frac{Re(L_\tau U_\tau^*)}{|L_\tau||U_\tau|} = -1 + \frac{1}{2}(\sin\phi\cot\theta X_\tau)^2 \tag{8.24}$$

$$\sin\Phi_\tau(LU) = \frac{Im(L_\tau U_\tau^*)}{|L_\tau||U_\tau|} = +\sin\phi\cot\theta X_\tau$$

$$\cos\Phi_\tau(US) = \frac{Re(U_\tau S_\tau^*)}{|U_\tau||S_\tau|} = -1 + \frac{1}{2}(\sin\phi\tan\theta X_\tau)^2 \tag{8.25}$$

$$\sin\Phi_\tau(US) = \frac{Im(U_\tau S_\tau^*)}{|U_\tau||S_\tau|} = +\sin\phi\tan\theta X_\tau$$

The results show small deviations from the unitary values that are in an excellent agreement with the experiment [17, 19] as shown in the Figures 2, 3 and 4. The consistency with experimentally determined phases [17, 19] requires that $\sin\phi < 0$. Note that the calculations of the sines are independent of the calculations of the cosines. The fact that the calculations are consistent with the approximations for small x of $\cos x = 1 - \frac{1}{2}x^2$ and $\sin x = x$ testifies to the self-consistency of the spin mixing mechanism and its predictions. The relative phases can, in fact, be solved exactly to read

$$\tan\Phi_\tau(LS) = \frac{Im(L_\tau S_\tau^*)}{Re(L_\tau S_\tau^*)} = \frac{-2\sin\phi X_\tau}{(1 + 2\cot 2\theta\cos\phi X_\tau - X_\tau^2)\sin 2\theta} \tag{8.26}$$

$$\tan\Phi_\tau(LU) = \frac{Im(L_\tau U_\tau^*)}{Re(L_\tau U_\tau^*)} = \frac{-\sin\phi\cot\theta X_\tau}{1 + \cot\theta\cos\phi X_\tau} \tag{8.27}$$

$$\tan\Phi_\tau(US) = \frac{Im(U_\tau S_\tau^*)}{Re(U_\tau S_\tau^*)} = \frac{-\sin\phi\tan\theta X_\tau}{1 - \tan\theta\cos\phi X_\tau} \tag{8.28}$$

We now turn our attention to the $f_0(980)$ mass region and $m \gtrsim 980$ MeV. We shall focus on amplitudes with transversity $\tau = d$. Since there is no change in the amplitudes with $\tau = u$ we assume we still have $\phi_1 = \phi_2 = \phi_3 = \phi$ and $\delta(U) = 2\phi$. With the reversed signs of L_d and U_d in (8.11) the spin mixing mechanism (8.13) reads

$$L_d = -e^{i\Phi(S_d)}e^{i\phi_1}\big(-\cos\theta|S_d^0| + e^{i\phi_2}\sin\theta|L_d^0|\big) \tag{8.29a}$$

$$S_d = -e^{i\Phi(S_d)}e^{i\phi_2}\big(+\sin\theta|S_d^0| + e^{i\phi_3}\cos\theta|L_d^0|\big) \tag{8.29b}$$

$$U_d = +e^{i\Phi(S_d)}e^{i\delta(U)}|U_d^0| \tag{8.29c}$$

The moduli have a form

$$\begin{aligned} |L_d|^2 &= \cos^2\theta|S_d^0|^2 + \sin^2\theta|L_d^0|^2 - \sin 2\theta\cos\phi|S_d^0||L_d^0| \\ |S_d|^2 &= \sin^2\theta|S_d^0|^2 + \cos^2\theta|L_d^0|^2 + \sin 2\theta\cos\phi|S_d^0||L_d^0| \\ |U_\tau|^2 &= |U_\tau^0|^2 \end{aligned} \tag{8.30}$$

Experimentally $|S_d|^2 \approx 0$ so that

$$|L_d|^2 \approx |L_d^0|^2 + |S_d^0|^2 \tag{8.31}$$

For $m \gtrsim 980$ MeV we expect $|L_d^0|^2$ small ≈ 0 and decreasing while $|S_d^0|^2$ is increasing as it emerges from the dip at 980 MeV. Then $|L_d|^2$ dips at 980 MeV and is increasing due to the mixing with the S-wave at $f_0(980)$ mass. This prediction is in agreement with the measured $|L_d|^2$[17, 19]. With the assumption $|L_d^0|^2 \approx 0$ the calculations of the phases also agree with the experiment provided there is a change of sign of $\sin\phi > 0$ in this mass region.

IX. CONCLUSIONS AND THE NATURE OF THE QUANTUM ENVIRONMENT.

The CERN measurements of $\pi^- p \to \pi^- \pi^+ n$ on polarized target at 17.2 GeV/c are so far the only high statistics measurements of its kind, yet they revealed an interesting new physics beyond the Standard Model. In Part I. of this work we have shown that they reveal a nonunitary interaction of the produced final state with a quantum environment. To preserve the identity of the final state particles this new interaction must be a pure dephasing interaction. In this Part II. we required that the dephasing interaction be fully consistent with the Standard Model. From this consistency alone we have deduced that in $\pi N \to \pi\pi N$ processes the new interaction with the environment must be a dipion spin mixing interaction. Applied to the S- and P-subsystem in $\pi^- p \to \pi^- \pi^+ n$ the theory predicts $\rho^0(770) - f_0(980)$ mixing. The predicted moduli and phases are in excellent qualitative agreement with the experimental results. Due to the selection rule $K = J-1, J, J+1$ the theory correctly predicts the absence of $f_2(1270) - f_0(1370)$ mixing. Since only $J = even$ are allowed in $\pi^- p \to \pi^0 \pi^0 n$ and in $\pi^+ p \to \pi^+ \pi^+ n$ there is no spin mixing in these processes, only a change of the phase of the S-matrix partial wave amplitudes.

The obvious question arises - what is the physical nature of the quantum environment? The consistency of the quantum environment with the Standard Model suggests that it is a universal environment in the entire Universe. It can be viewed as a sea of quantum states $\rho(E)$ across the Universe. The quantum states $\rho(E)$ must be stable. Mathematically they are equivalent to two-qubit quantum states [25]. They are either entangled or separable two-qubit states forming two distinct subsystems. To protect their entanglement or separability the quantum states $\rho(E)$ cannot interact with any of the particles of the Standard Model including the Higgs boson.

The pure dephasing interaction involves only the exchange of quantum information or quantum entanglement between the two quantum states $\rho_f(S)$ and $\rho_i(E)$. As such it stands outside of the Standard Model. While the dephasing interaction can have strong observable effects, such as $\rho^0(770) - f_0(980)$ mixing, its effects are not observable in the usual studies of particle scattering. Since these experiments do not involve diparticle partial wave amplitudes in measurements on polarized targets, they cannot observe the resonance mixing. The measurements of relative phases alone cannot provide evidence for the quantum environment since the needed unitary phases are known only for $\pi N \to \pi\pi N$ processes. The dephasing interaction is largely a "dark" interaction.

The universal presence of the quantum environment and its pure dephasing interaction with the baryonic matter must manifest themselves in astrophysical observations. Such observations provide a convincing evidence for the existence of dark matter and dark energy which are both omnipresent environments in the Universe with non-standard interactions with baryonic matter. It is our conjecture that the quantum environment can be identified as the common origin of dark matter and dark energy whereby the entangled states $\rho(E)$ constitute dark energy and the separable states form dark matter. The entanglement of dark energy gives rise to the pressure of the cosmic fluid. These conjectures suggest that the observation of $\rho^0(770) - f_0(980)$ mixing represents the first laboratory detection of the dark Universe.

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